In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institution shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

OPTIMUM INSPECTION AND MAINTENANCE INTERVALS FOR PROCESSES SUBJECT TO CHANCE AND WEAROUT FAILURES

A THESIS

Presented to

The Faculty of the Graduate Division

by

James Joseph Buckley

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Industrial Engineering

Georgia Institute of Technology

June, 1963

OPTIMUM INSPECTION AND MAINTENANCE INTERVALS

FOR PROCESSES SUBJECT TO CHANCE

AND WEAROUT FAILURES

Approved:

Date Approved by Chairman: June 28, 1963

ACKNOWLEDGMENTS

The author wishes to express grateful appreciation to all who have helped make this undertaking successfully completed.

Thanks are especially due to his parents for their unquestioning support throughout this study.

In particular he extends thanks to Professor J. J. Moder for his interest and guidance as thesis advisor. Dr. Moder conceived the idea of this study and his many suggestions were invaluable. Special credit and thanks are due to Dr. J. H. MacKay for his criticisms, encouragement, and suggestions in the mathematical developments, criterion function, and text. Dr. H. M. Wadsworth also made valuable suggestions on the manuscript.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

LIST OF TABLES

LIST OF TABLES (Continued)

LIST OF ILLUSTRATIONS

SUMMARY

The object of this study is to consider relevant process costs in formulating a criterion for the economical design of quality control procedures for a general class of processes. The criterion of maximizing net income is used in this study.

The purpose of this study is to measure the average net income for a process where process quality is a deterministic variable (defectives are continuously produced once a failure has occurred) or a stochastic variable. If process quality is a deterministic variable this study

(1) determines the production inspection interval when process failure is a result of chance causes,

(2) determines a procedure to find the production inspection interval and preventive maintenance interval when process failure is a result of wearout and chance causes,

so that the expected net income is maximum. If process quality is a stochastic variable then the purpose is to outline a method of determining the inspection and preventive maintenance intervals, sample size, and other quality control variables when process failure is a result of wearout and chance causes, so that expected net income is maximum.

In the first production system investigated process failure is a result of chance causes and process quality is a deterministic variable. The expected net income equation is developed for eight categories. The categories are combinations of the following three divisions:

(1) screening damages the product or screening does not damage the product,

(2) total production for the year is variable or total production for the year is a predetermined constant,

(3) sequential or continuous screening of past production for defectives is used.

When total production is fixed the criterion reduces itself to minimizing costs. The optimum inspection interval is calculated for various selections of the risk and cost parameters when total production is variable and continuous screening is employed, and when production is fixed and either sequential or continuous screening is used. Approximate expressions for the optimum inspection interval are also derived. The same general results apply if screening damages the product and/or there are n mutually independent failure components. It is shown that the optimum inspection interval is greater when sequential search is used in place of continuous screening if total production is fixed. The optimum inspection interval is usually smaller when income is maximized then if costs are minimized. Maximum expected net income is larger when sequential search is employed than if continuous screening is used.

The second process studied incorporates both wearout and chance failures with preventive maintenance and process quality is a deterministic variable. The gamma probability law is used to describe the phenomenon of wearout. The maintenance plan is to overhaul the process completely whenever it fails as a result of wearout or at the automatic preventive maintenance shutdown time. Only the case where screening does not damage the product in any way is considered and sequential search for defectives is employed. No solutions are given but the expected net income equation is derived for total production fixed or variable.

The third process studied extends the second process to the case where process quality is a stochastic variable. The selection of a "quality control plan" (i.e. a $\overline{X}$-chart or p-chart, etc.) and the application of the criterion are discussed but no solutions are given.

It is suggested that simulation be used in locating the optimum operating conditions for the second and third processes studied.

CHAPTER I

INTRODUCTION

Definition of the Problem and the Scope of the Study

At the present time the majority of quality control plans are based upon statistical criteria without reference to the relevant process costs. The object of this study is to consider the latter in formulating a criterion for the economical design of quality control procedures for a general class of processes.

The basic assumptions that outline the system under study are:

(1) the process may have any number of components whose failure causes defective production, and component failures are mutually independent,

(2) process failure is a result of independently occurring chance or wearout causes,

(3) when a process failure occurs production immediately shifts from zero per cent defective to one hundred per cent defective,

(4) the production rate is constant and only discrete products are considered,

(5) sequential or continuous screening of past production for defectives may be used,

(6) the screening activity may or may not damage the product,

(7) the number of units produced during the period of interest may be fixed or variable,

(8) the various risk, cost, and income parameters are known, or at least estimates are obtainable, and

(9) the long range point of view is adopted.

The first model studied assumes that failures are only a result of chance causes. Numerical examples for this model are studied to see how variation in the various risk and cost factors affect the optimum design. The second model investigated incorporates wearout with preventive maintenance, along with chance failures. Finally, the third assumption above is relaxed, to allow for drifts and/or shifts in process quality.

The Criterion

One of the earliest applications of statistical theory to production quality control was the original Shewhart control chart. This chart evolved through its various forms to the present day V-mask cumulative sum chart. This progress is of central interest to quality control, but the question this study asks, and answers, is "What considerations, besides statistical ones, should determine the values of the quality control plan variables?"

In many industries, namely missiles and electronics, where quality or reliability is of paramount importance, quality control costs are rapidly increasing. Feigenbaum (6) has presented a typical example where quality control costs amounted to seven per cent of the sales dollar. Therefore, the first consideration, and the only one usually applied in practice, would be to reduce quality control costs while maintaining present quality levels. This type of economical design will many times result in large savings, but it is a sub-optimization since only quality control costs are thought relevant.

What is needed is a theoretical argument that states what the control plan variables should be for optimum operating conditions. The optimum operating condition that will have the greatest appeal to business would be maximum net income. This criterion will incorporate such items as the product sale price, total production costs, the number of units to be produced, etc., and is only a sub-optimization insofar as the process under study may be a part of a larger production system.(*) When the total number of units produced is fixed, then this criterion will reduce to the usual one of minimizing total relevant cost. The criterion of maximizing net income is used in this study.

Purpose

The purpose of this study is to establish a criterion that measures the average net income for a process where process quality is either a deterministic(**) or stochastic variable. If process quality is a deterministic variable, the purpose is:

(i) to determine the production inspection interval when process failure is a result of chance causes, and

(ii) to determine a procedure to find the production inspection interval and the interval of preventive maintenance when process failure is a result of wearout and chance causes, so that the average net income is maximum.

If process quality is a stochastic variable, the purpose is to outline a method to determine the inspection and preventive maintenance

---

(*) In this case individual process studies might be combined into an overall optimization study.

(**) That is, a process which continuously produces defectives once a failure has occurred.

interval, sample size, and other quality control variables when process failure is a result of wearout and chance causes, so that the average net income is maximum.

Other Research in Economical Quality Control Plan Design

The economical design of quality control plans is receiving an ever increasing amount of attention. The type of process that is most widely studied, because it is well suited to statistical analysis and is of practical importance, is where the process mean shifts a fixed distance whenever a failure occurs. Previous studies, Page (7,8), Weiler (3,4,5), and Aroian (2), have sought the optimum sample size for control charts by minimizing the average amount of inspection required to detect the shift in the process quality.

Duncan (1) determines the control chart limits, sample size and the inspection intervals for maximum average net income for a process that fails as a result of chance causes. Ito (9) studies the selection of the same variables for minimizing the relevant process quality costs.

There have only been a few studies (1,9,31) where the criterion of maximizing income, or minimizing costs, has been applied to quality control plan design. To the author's knowledge, neither criterion has been applied to processes that fail as a result of both wearout and chance causes.

CHAPTER II

MODEL I

Introduction

In this chapter a life-death manufacturing process is studied where, as a result of chance causes,(*) the product fraction defective changes from zero per cent to one hundred per cent during an infinitesimal period of time. A criterion is developed to select the intervals of time at which production is inspected so that the expected net income for the period of time y is at its maximum value. It is shown that if the plant has excess production capacity the above criterion reduces itself to finding the inspection interval that makes the expected total variable cost per non-defective unit produced a minimum. The model incorporates the possibilities for "sequential" or continuous screening of past production for defective units. It is assumed that:

(1) the screening and inspection activities do not damage the product in anyway,

(2) no defectives are sold, or that the screening procedure will eliminate all defectives, and

(3) the process is stopped only when a failure is discovered.

Model I is developed in Appendix E for the case when the screening and inspection activity cause the product to become defective.

(*) Process failure as a result of chance causes means that process failures occur at random intervals.

The process may contain any number of components whose failure results in defective production. The restriction on process failures is that each component fails as a result of independently occurring chance causes and component failures are mutually independent.

Although it is true that wearout failure is always present, if it is assumed that the mean time between wearout failures is ten or more times the order of the mean time between chance failures, then the effect of wearout failure will be small.(*) They will be neglected in this chapter. Infant mortality, or the phenomena of initial and very early machine tool failure, is not considered in Model I. A model with only chance failures is the simplest failure model and therefore will be studied first.

A system with wearout components will progress to a state where all failures occur randomly in time when there is no preventive maintenance (see (30)). Model I applies to this case if one waits for the system to get "old."

Examples

(1) Consider an automatic machine, such as an automatic punch press or cutting tool, that produces units at a constant rate. The punch press or cutting tool may slip out of line, chip, break, or otherwise fail because of independently occurring chance causes. The problem is to find at what intervals the machine's production should be checked so that maximum income is obtained. The machine considered may actually be

---

(*) More specifically if $\alpha > 4$, $\bar{\mu}_w \geq \alpha\bar{\mu}_c = \alpha\lambda^{-1}$, and $\sigma_w < (\alpha-4)/(\lambda)$. (2.3), where the w subscript refers to the wearout distribution, then it is assumed that the effect of wearout failure can be neglected.

a group of machines in series where production is only inspected after the unit has passed through the last operation.

(2) Consider a missile standing in its silo where, from time to time, it is checked out to see if it will respond correctly to a firing impulse. The missile fails, or does not fire when called upon to do so, as a result of independently occurring chance causes such as erosion and corrosion. Naturally, the military would like the fraction of ready missiles (i.e., those missiles that will fire) per total missile population at this base to be maximum, but costs can not be ignored. A possible criterion might be to use the inspection interval that makes the per cent of ready missiles per total operational cost a maximum.

(3) On the more general level, consider a certain land area L through the passage of time. The system is said to fail when T per cent of the inhabitants of L have characteristic C. To be more specific, let L be a geographical area in a particular country and C be malaria. Assuming T =.0 at the start, or that there has just been a successful malaria eradication campaign in L, the arrivals of C in L may be random. The problem is to decide on the time intervals of malaria inspection in L such that the per cent of malaria free inhabitants in L per total operational cost is maximum.

The Problem

A general type of manufacturing process is investigated and it is assumed that the production rate is a constant number, R, of discrete units per process running hour. The units are usually one hundred per cent acceptable from the quality standpoint; however, occasionally the process produces units which are one hundred per cent unacceptable.

When unacceptable units are being produced it shall be said that the process, or equivalently the machine tool, has failed. When a failure occurs, production immediately changes from zero per cent defective to one hundred per cent defective and the process will continuously produce defectives until the failure is discovered and the machine tool is fixed.

Assume the process contains n components whose failure results in defective production. The arrival rate of process failures, as a result of chance causes, is considered to be at an average rate of λ occurrences per failure free running hour.

The quality control procedure is to inspect the process every h process running hours and if the observed item is acceptable let the process continue operating. If the observed item is defective then the inspector immediately stops production, the machine tool is put back into a non-failed state, and past production is screened for defectives.

The problem is first to develop the criterion of maximizing net income in terms of h and then to calculate the optimum h for a suitable range of the various cost and risk parameters.

Process Failure Density Function

The failure density function for a process that fails as a result of chance causes is assumed to be

$$e(t) = \begin{cases} \lambda e^{-\lambda t} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}, \qquad (1)$$

where λ^{-1} = the mean time between process failures.

This result is derived in Appendix A for a process that has n failure components. The failure density function is the same as equation (1) when the process has one of n failure components. The change occurs in the process failure arrival rate and λ will be the sum of all the component's failure arrival rates. Model I is generalized to the case of a process with n failure components in Appendix A. In the rest of this chapter our attention will be restricted to a process that has one component whose failure results in defective production and the failure is a result of one chance cause.(*) The estimation of the parameter λ is discussed in Appendix B.

Defining the Criterion

Define the number T_M to be

$$T_M = \underset{h \geq 0}{\text{Maximum}} \left[\text{Expected} \left\{ \frac{\text{net income}}{\text{y years}} \right\} \right], \tag{2}$$

where h is the number of process running hours between inspections.

Assuming the number T_M exists,(**) let h_{op} be that value of $h \geq 0$ which will produce the number T_M. When $h = h_{op}$ the expected net income for the period y will be maximum. Equation (2) will be used as the criterion for selecting a value of h.

It is easy to show that the value of h_{op} is independent of y.(***) Therefore, without loss of generality, y will be assumed to be unity throughout the remainder of this study.

(*)Appendix A also incorporates the possibility of many independently occurring causes of chance failure for each of the n process failure components.

(**)It is shown in later sections that in any practical case the number T_M exists and that h_{op} is usually unique.

(***)There is one restriction on y and that is that it must be large enough to constitute a "long range" study. This restriction is explained in a later section.

Terminology

All the following terms are defined so that the criterion function may be expressed in terms of its variable h. All the symbols, and terms, used in this chapter are presented in the List of Symbols in Appendix L for easy reference.

Let E be the statistical expected value operator,

h = the inspection interval in process running hours,

$[\![\;]\!]$ denote the greatest integer function, which is defined as:

$$[\![X]\!] = n \quad \text{if } n \leq X < n + 1, \text{ and}$$

n is an integer, X, any real number,

V = the dollar sale price of a unit of production,

U = the total number of saleable units produced during the year,

H_R = the total clock hours per year that the plant is in operation,

R = the production rate in the number of discrete units produced per process running hour (assumed constant),

λ = the mean number of process failures, as a result of chance causes, per failure free process running hour,

s = the dollar income received from a defective unit (which may be obtained as a result of selling defectives for scrap or from the re-working of defectives for normal sale; this is usually called salvage value and $0 \leq s < V$),

C = the cost in dollars for the inspector to inspect the process to see if it has failed or not failed,

S = the cost in dollars per unit to screen production in order to classify each item as defective or not defective, "Tool cycle" = the elapsed clock hours from when the process begins producing non-defective units until the process is ready to start again with a machine tool in the non-failed state after a failure has been discovered and remedied,

R_C = the retooling cost, or the cost in dollars of getting the machine tool into a non-failed state after a failure has been discovered,

D = the clock hours of no production per tool cycle (this is the time it takes to get the process ready to begin production again after a failure has been discovered and includes the screening and retooling activities; this number is usually termed the "downtime"),

L = the process running hours per tool cycle,

I = the number of inspections per tool cycle,

H_b = the number of process running hours that the process is making defectives per tool cycle,

N_o = the number of units that must be screened per tool cycle to eliminate all defectives,

T = the process running hours to a failure, and

$$E(L) = \bar{L}\ ,\ E(I) = \bar{I},\ E(H_b) = \bar{H}_b,\ E(N_o) = \bar{N}_o\ ,$$

v = the total variable cost per non-defective unit produced,

Y = the total variable production cost per unit produced, (which includes such costs as materials, labor, power, etc., and note that $Y < v$),

d = the mean clock hours it takes the inspector to decide whether the process has failed or not failed (assume $0 \leq d < h$),

f_x = the total fixed process cost for the year (including such items as rent, taxes, depreciation, etc.),

C_d = the cost in dollars for each hour of idle time when the process should be in production (this number comprises all those costs that are not eliminated when the process is not in production, for example, idle labor costs).

All the parameters defined above are positive non-zero (except s, d and possibly C_d) quantities.

If the process under study is a part of a larger production line, then V will be the value of the product at this point in production.(*) In Model I, D will be taken as a constant. Hence, it is assumed that the screening time is either less than the retooling time or else that it can be conducted independently from the process. It is further assumed that the inspector inspects the last item produced. Unless otherwise stated, U is assumed to be a variable.

Development of the Criterion Function

It is seen that

$$E\left\{\frac{\text{net income}}{\text{year}}\right\} = E\left\{U\left[\frac{\text{net income}}{\text{unit sold}}\right]\right\} =$$

$$= E\left\{U\left[V - \left\{v + \frac{f_x}{U}\right\}\right]\right\} = VE(U) - E(Uv) - f_x \ . \qquad (3)$$

Derivation of E(U)

Let

T_i = the process running hours to a failure in the ith tool cycle,

L_i = the process running hours to when a failure is discovered for the ith tool cycle.

(*)It will be seen that h_{op} is only dependent upon λ, Y, s, S, d, R, and C if U is constant and h_{op} is dependent upon d, D, V, C_d, R_c, λ, Y, s, S, R, and C if U is a variable. Therefore, if U is fixed, only estimates for λ, Y, s, S, d, R, and C need to be obtained in order to calculate h_{op}.

Define

$$\overline{F} = \frac{\text{total failure free process running hours}}{\text{plant operating clock hour}} \,,$$

and if m is the total number of tool cycles in the year, then

$$\overline{F} = \frac{\sum_{i=1}^{m} T_i}{\sum_{i=1}^{m} (L_i + D)} \,. \tag{4}$$

It can be shown, see (22) or (23), that

$$\lim_{m \to \infty} E(\overline{F}) = \frac{E(T_i)}{E(L_i) + D} \,, \tag{5}$$

for any $i = 1,2,\ldots$. Assuming H_R is large, so that m is large, the following approximation for $E(\overline{F})$ will be used(*)

$$E(\overline{F}) \doteq \frac{E(T_i)}{E(L_i) + D} \,. \tag{6}$$

It follows that

$$E(T_i) = \int_0^\infty \lambda t e^{-\lambda t} dt = \lambda^{-1} \,, \tag{7}$$

(*) This approximation is expressed in terms of m in Appendix C. It is assumed throughout this study that the long range point of view is adopted. This implies that m is large and hence the approximation of equation (6) is useful. This same reasoning is applied a number of times in this study. If one wishes to apply the developments to a short range study, then an analysis similar to that of Appendix C should be carried out each time this type of approximation is used.

for any tool cycle,

Therefore,

$$U = H_R \, R \, \overline{F} \, ,$$

hence

$$E(U) \doteq \frac{H_R \, R}{\lambda(\overline{L} + D)} \, . \tag{8}$$

From Appendix D,

$$\overline{L} = h\overline{I} + d \, , \tag{9}$$

and it now follows that

$$E(U) \doteq \frac{H_R \, R}{\lambda(h\overline{I} + d + D)} \, . \tag{10}$$

<u>Derivation of E(v)</u>

Let

M_i = the total variable cost incurred in the ith tool cycle,

T_i = the process running hours to a failure in the ith tool cycle,

L_i = the total process running hours in the ith tool cycle,

I_i = the total number of inspections in the ith tool cycle,

H_{bi} = the amount of time the process is producing defectives in the ith tool cycle, and

N_{oi} = the number of units screened in the ith tool cycle.

The total variable cost, M_i, for the ith tool cycle, will be composed of the sum (inspection cost = CI_i) + (screening cost = SN_{oi}) +

(defective production {salvage income} cost = $-sRH_{bi}$) +

(downtime costs = $DC_d + R_C$) +

(total variable production cost = YRL_i) . (11)

Therefore

$$M_i = CI_i = + SN_{oi} + DC_d + R_C - sRH_{bi} + YRL_i \ ,$$

or

$$M_i = CI_i + SN_{oi} + R(Y-s)H_{bi} + DC_d + R_C + YRT_i \ , \tag{12}$$

since

$$L_i = H_{bi} + T_i \ . \tag{13}$$

Define

$$\overline{O} = \frac{\text{total variable cost}}{\text{non-defective unit produced}} \ ,$$

and if m is the total number of tool cycles in the year, then

$$\overline{O} = \frac{\sum_{i=1}^{m} M_i}{\sum_{i=1}^{m} RT_i} \ .$$

It can be shown, see (22) or (23), that

$$\lim_{m \to \infty} E(\overline{O}) = \frac{E(M_i)}{RE(T_i)} \ ,$$

for any i = 1,2,... . Assuming H_R is large, so that m is large, the following approximation for $E(\overline{O})$ will be used(*)

$$E(\overline{O}) \doteq \frac{E(M_i)}{RE(T_i)} . \tag{14}$$

Now, for any tool cycle

$$E(M_i) = C\overline{I} + R(Y-s)\,\overline{H}_b + S\overline{N}_o + C_d D + R_C + YR\lambda^{-1} , \tag{15}$$

since, from equation (7), $E(T_i) = \lambda^{-1}$.

Substituting equations (7) and (15) into equation (14) it is seen that

$$E(v) = E(\overline{O}) \doteq \frac{\lambda}{R}\,(C\overline{I} + R(Y-s)\overline{H}_b + S\overline{N}_o + C_d D + R_C) + Y . \tag{16}$$

The expressions for $\overline{I}$ and $\overline{H}_b$ are derived in Appendix D.

Derivation of $E(Uv)$

It is easily seen that

$$Uv = \text{the total variable cost for the year.}$$

Define

$$\overline{F}_o = \frac{\text{the total variable cost}}{\text{plant operating clock hour}} ,$$

and then

$$\overline{F}_o = \frac{\sum_{i=1}^{m} M_i}{\sum_{i=1}^{m} (L_i + D)} ,$$

(*)See the footnote on page 13.

since m is the total number of tool cycles in the year.

Assuming H_R is large, so that m is large, the following approximation for $E(\overline{F}_o)$ will be used(*)

$$E(\overline{F}_o) \doteq \frac{E(M_i)}{E(L_i) + D} \quad .$$

Now

$$Uv = H_R \overline{F}_o \ ,$$

hence

$$E(Uv) \doteq \frac{H_R E(M_i)}{E(L_i) + D} \ , \tag{17}$$

for any $i = 1, 2, \ldots, m$. Using equation (15) for $E(M_i)$ and equation (9) for $E(L_i)$ in equation (17) it is seen that

$$E(Uv) \doteq H_R \left\{ \frac{C\overline{I} + R(Y-s)\overline{H}_b + S\overline{N}_o + C_d D + R_C + YR\lambda^{-1}}{h\overline{I} + d + D} \right\} . \tag{18}$$

<u>The Criterion Function</u>

Substituting equations (10) and (18) into equation (3) it follows that the criterion function becomes

$$T_M \doteq \underset{h \geq 0}{\text{Max}} \left\{ \frac{(V-Y)R\lambda^{-1} - \left[C\overline{I} + R(Y-s)\overline{H}_b + S\overline{N}_o + C_d D + R_C \right]}{h\overline{I} + d + D} \right\} H_R - f_x . \tag{19}$$

Therefore, h_{op} is determined by the number D_M where

$$D_M = -(T_M + f_x) / H_R \ ,$$

(*)See footnote on page 13.

and it follows that

$$D_M \doteq \underset{h \geq 0}{\text{Minimum}} \left\{ \frac{C\bar{I} + R(Y-s)\bar{H}_b + S\bar{N}_o - C_S}{h\bar{I} + d + D} \right\} , \tag{20}$$

where

$$C_S = (V-Y)R\lambda^{-1} - (DC_d + R_c) > 0 . \tag{21}$$

Only the expressions for $\bar{I}$, $\bar{H}_b$, and $\bar{N}_o$ will contain the variable h. This implies, if $h_{op} \neq 0$ or $h_{op} \neq \infty$, that equation (20) might be differentiated with respect to h and then solved for h_{op}. This will be the procedure once expressions for $\bar{N}_o$ are determined.

Expressions for $\bar{N}_o$

If the time sequential order of all the units of doubtfully acceptable past product is determinable then the screening procedure may be continuous or sequential. If the time sequential order of past production is not determinable, for example if the units of production go directly into a totebox, then all doubtfully acceptable past production must be screened. In the latter case

$$\bar{N}_o = R(h + d) - 1 . \tag{22}$$

Continuous Screening

In this case it is better to screen from the first unit produced in the inspection interval until the first defective unit is found. The screening is forward in time because with the exponential failure distribution the expected time to a failure is always less than $h/2$, if a failure has occurred within the inspection interval. Therefore

$$\bar{N}_o = R\left\{(h + d) - \bar{H}_b\right\} + 1 . \tag{23}$$

Sequential Screening

A sequential search procedure is one in which the choice of the $(k + 1)$th inspected item is a function of the observation, defective or non-defective, on the kth inspected item. The optimum sequential screening plan will be that scheme that minimizes the mean number of inspected units needed to find all the defective items. Moder (16) shows that a good approximation to this optimum plan for the exponential distribution is the intuitive rule of selecting the unit at the mid-range of the resulting interval of uncertain production for the next inspection. For all practical purposes, the mid-range plan is optimum for the exponential distribution.(*)

$\bar{N}_o$ for the Mid-range Sequential Screening Plan

Define

$$f_1(h; R,d) = \left[\!\left[\frac{R(h + d) - 2}{2}\right]\!\right] , \tag{24}$$

$$f_2(h; R,d) = \left[\!\left[\frac{f_1(h; R, d) - 1}{2}\right]\!\right] , \tag{25}$$

and in general

$$f_{n+1}(h;R,d) = \left[\!\left[\frac{f_n(h; R, d) - 1}{2}\right]\!\right] , \tag{26}$$

(*) If $\lambda h < 1$, a condition that holds in any practical case, the maximum difference between the expected value of the remaining interval of uncertain production when the optimum sequential screening plan is used and when the mid-range selection is used is 0.007 of an hour when $\lambda = h = 1$ (see (16)). This maximum difference of (0.007)Rh units of production is negligible because numerical studies suggest that Rh_{op} will be less than 1000.

for $n = 1,2,3,\ldots$ and $R(d+h) \geq 2$.

It is easily seen that there is a n^* such that

$$f_{n^*}(h;R,d) = 0 \,. \tag{27}$$

It is asserted that

$$n^* \leq \overline{N}_o \leq n^* + 1 \,. \tag{28}$$

Initially, after a failure has been discovered, there are $R(h+d)-1$ units of doubtful production. When the kth unit is to be screened, the smaller area of the two possible resulting areas of doubtful production will contain $f_k(h;\, R,d)$ units.(*) Repeating this procedure for $k = 1,2,\ldots$ until $k = n^*$ and equation (27) holds, the assertion $\overline{N}_o \geq n^*$ is shown. It is easily seen that N_o is a random variable that will equal n^* or $n^* + 1$. Therefore, the assertion $E(N_o) = \overline{N}_o \leq n^* + 1$ is shown.

In order to determine n^* replace all the brackets denoting the greatest integer function in equation (27) by ordinary brackets. If $\overline{n}$ is the largest positive integer such that

$$\frac{R(h+d) - 2}{2^{\overline{n}}} - \sum_{j=1}^{\overline{n}-1} 2^{-j} \geq 0 \,,$$

or

$$\overline{n} \leq \log \left\{ R(h+d) \right\} / \log 2,$$

then

(*)Consider the units to be lying along the time axis with time increasing towards the right. If there are an even number of units in an area of doubtful production then, because of the exponential failure density function, the unit to the left of the midpoint should be selected to be screened next.

$$n^* = \left[\left[\frac{\log \left\{R(h+d)\right\}}{\log 2}\right]\right].$$

Therefore

$$\overline{N}_o = \left[\left[\frac{\log \left\{R(h+d)\right\}}{\log 2}\right]\right] + \epsilon , \tag{29}$$

where

$$0 \leq \epsilon \leq 1 .$$

It is noticed that if

$$h = \frac{2^k}{R} - d \text{ , for any } k = 1,2,3,\ldots ,$$

then $\overline{N}_o = k$ and $\epsilon = 0$. When h is close to the point $(2^k/R) - d$ from the left, then ϵ is close to one. Studies of the values of $\overline{N}_o$ for various selections of R(h+d) suggests that ϵ is approximately a linear function of h. The linear approximation for ϵ is found to be(*)

$$\epsilon \doteq R(h+d)2^{-\left[\left[\frac{\log \left\{R(h+d)\right\}}{\log 2}\right]\right]} - 1. \tag{30}$$

Setting ϵ to be a linear function of h is convenient at this point

---

(*)The equation for a straight line between the points $(h = (2^n/R)-d,$ $\overline{N}_o = n)$, $(h = (2^{n+1}/R) - d,\ \overline{N}_o = n+1)$, where

$$n = \left[\left[\frac{\log \left\{R(h+d)\right\}}{\log 2}\right]\right],$$

is $n - 1 + R(h+d)2^{-n}$, from which equation (30) is deduced.

because $\overline{N}_o$ will be a continuous function of h.(*) Using equation (30) in equation (29), it follows that

$$\frac{d}{dh}\overline{N}_o \doteq R\,2^{-\left[\left[\frac{\log\{R(h+d)\}}{\log 2}\right]\right]},$$

if h is suitably restricted. It is seen that if

$$h \notin J = \left\{ \frac{2^k}{R} - d \;\middle|\; k = 1,2,\ldots. \right\}, \tag{31}$$

then $\overline{N}_o$ is differentiable with respect to h and the above relation for $d\overline{N}_o/dh$ is valid.

Define the function $\rho(\quad)$ to be

$$\rho\,(R(h+d)) = \left[\left[\frac{\log\{R(h+d)\}}{\log 2}\right]\right] - 1 + R(h+d)2^{-\left[\left[\frac{\log\{R(h+d)\}}{\log 2}\right]\right]}. \tag{32}$$

The approximation

$$\overline{N}_o \doteq \rho(R(h+d))$$

will be used for the mid-range sequential screening plan.

<u>The Two Extreme Cases of the Criterion</u>

If the plant is operating at full capacity during the year, then U

(*)The other possibilities are to set ϵ to be a constant, or to derive the exact expression for ϵ. Having ϵ constant makes the income equation discontinuous and h_{op} will usually be one of the points

$$(2^k/R) - d - \delta,\quad k = 1,2,\ldots\,,$$

where $\delta > 0$ is infinitesimally small -- which is inaccurate and unrealistic. The added accuracy of using the exact expression for ϵ is not warranted in this study.

will be a function of h. At the other extreme consider the plant to have considerable excess capacity so that U is a constant dictated by demand. When U is a variable the criterion for selecting h is as stated and developed in the preceding sections. If U, however, is a constant, then the criterion for finding h may be simplified further.

Development of the Criterion - U Constant

In this case, referring to equation (3), it is seen that

$$T_M = \underset{h \geq 0}{\text{Max}} \left\{ VU - UE(v) \right\} - f_x \;.$$

Therefore, h_{op} is determined by the number G_M, where

$$G_M = \underset{h \geq 0}{\text{Minimum}} \; E(v) \;. \tag{33}$$

Using equation (16) for E(v) in equation (33) it follows that

$$G_M \doteq \underset{h \geq 0}{\text{Min}} \left\{ \frac{\lambda}{R} \left[C\overline{I} + R(Y-s)\overline{H}_b + S\overline{N}_o + R_C + C_d D \right] + Y \right\}, \tag{34}$$

or

$$G_M \doteq \frac{\lambda}{R} \left[\underset{h \geq 0}{\text{Min}} \left\{ C\overline{I} + R(Y-s)\overline{H}_b + S\overline{N}_o \right\} + R_C + C_d D \right] + Y \;. \tag{35}$$

When U is constant the criterion reduces itself to minimizing the total variable cost per non-defective unit produced. In this case h_{op} is independent of U, V, H_R, R_C, D, C_d, and f_x. The expressions for $\overline{I}$ $\overline{H}_b$, and $\overline{N}_o$ are functions of h, and $\overline{I}$ and $\overline{H}_b$ are derived in Appendix D.

Nomenclature - Cases to be Studied

Let(*)

TRI = the total relevant income equation,

TRC = the total relevant cost equation, then

$$TRI_i = \frac{C\overline{I} + R(Y-s)\overline{H}_b + S\overline{N}_{oi} - C_S}{h\overline{I} + d + D} \quad , \tag{36}$$

$$TRC_i = C\overline{I} + R(Y-s)\,\overline{H}_b + S\overline{N}_{oi} \quad , \tag{37}$$

where i = 1 means continuous screening is used and $\overline{N}_{o1}$ is defined by equation (23), and

i = 2 means sequential mid-range screening is used and $\overline{N}_{o2}$ is defined by equation (32).

The TRI equation is applicable when U is a variable (see equation (20)) and the TRC equation is applicable when U is a constant (see equation (35)).

It can be shown that on the average sequential mid-range search will screen less units per process failure than continuous screening, if(**) $\lambda h \leq 1$

and

(1) d = 0, or

(*) When complete screening is used, or $\overline{N}_o = R(h+d) - 1$, it is easily seen that the solution for h_{op} is contained in the general solution for i = 1. This case will not be studied further.

(**) This result follows from:
(i) expanding $\overline{H}_b$ in an alternating series (see (1) and (17)), and
(ii) then noting

$$\overline{N}_o > 5Rh/12 + 1 \quad ,$$

when continuous screening is used.

(2) $d = h/2$ and $Rh \geq 3$, or

(3) $d = h$ and $Rh \geq 6$.

If all the parameters and h are kept constant, a decrease in $\overline{N}_o$ will produce a corresponding increase in the expected net income. Therefore, in any practical case, sequential mid-range search should be used when the time sequential order of doubtfully acceptable past production is determinable.

The case when $i = 1$ is still of interest because:

(1) The general solution for $i = 1$ is the general solution when continuous screening is employed and the screening activity damages the product (see Appendix E) or when complete screening is used,

(2) h_{op} for $i = 1$ may be used to locate h_{op} when $i = 2$.

The TRI Equations

The TRI_1 Equation - Continuous Screening

Substituting the expression for $\overline{N}_o$, equation (23), into equation (36) reveals that the criterion of maximizing expected net income results in locating the positive zeros of

$$\frac{d}{dh}\left\{\frac{C\overline{I} + R(Y-s)\overline{H}_b + S(R\left\{(h+d) - \overline{H}_b\right\} + 1) - C_S}{h\overline{I} + d + D}\right\} = 0. \quad (38)$$

Using equation (6) of Appendix D for $\overline{H}_b$, then after differentiation and simplification equation (38) becomes

$$(C_{S1} + \alpha_o - B)e^{+\lambda h} + \alpha_o(1-A)e^{-\lambda h} - (\alpha_o A + C_{S1})\lambda h +$$

$$(1-A)\lambda h^2 - (\lambda\alpha_o B + C_{S1} - \alpha_o A + 2\alpha_o) = 0 , \quad (39)$$

where[*]

$$A = \frac{(Y-s) - S}{Y-s} \geq 0 , \tag{40}$$

$$B = \frac{C}{R(Y-s)} > 0 , \tag{41}$$

$$\alpha_o = d + D , \tag{42}$$

$$C_{S1} = \frac{A}{\lambda} + \frac{(V-Y)R\lambda^{-1} - (C_d D + R_C + S)}{R(Y-s)} - d > 0 . \tag{43}$$

If $(C_{S1} + \alpha_o - B) > 0$,[**] then it is seen that equation (39) is satisfied for a unique $h > 0$.[***] This zero will be the h coordinate at the minimum of the TRI_1 equation since

$$\lim_{h \to 0} TRI_1 = + \infty ,$$

and

$$\lim_{h \to \infty} TRI_1 = R(Y-s) .$$

Therefore, h_{op} will be the positive zero of equation (39).

<u>Solution of the TRI_1 Equation.</u> Equation (39) was solved for its positive zero for various values of the parameters A, B, λ, α_o, and C_{S1}

(*) It has been assumed that $s < V$ and it is now assumed that $s < Y$, and $1 \geq A \geq 0$.

(**) If $(C_{S1} + \alpha_o - B) < 0$ then it follows (see equation (20)) that $T_M < 0$, since $Y > s$. In any practical case the expected net income will be positive for $h \doteq h_{op}$, therefore $(C_{S1} + \alpha_o - B) > 0$.

(***) This follows by first taking the second degree polynomial in h to the right hand side of equation (39). When $h = 0$ the right side is greater than the left side and as h increases the polynomial decreases and the exponential function on the left side increases. Therefore, the two functions can be equal for only one $h > 0$.

and Tables 1 through 11, and Figures 5 and 6, in Appendix F display the results. Only positive selections for A were used because a negative A, or $S > (Y-s)$, would be of little practical importance. The variation in the parameters were chosen so as to cover the region of most practical applications.

The positive zero of equation (39) was located by use of the Burroughs 220 Digital Computer at the Rich Electronic Computer Center at Georgia Institute of Technology. The results are accurate to all decimal places shown in the tables.

Variation in TRI_1 for Changes in h about h_{op}. Using equation (36) for TRI_1, let TRI_1^* be defined as

$$TRI_1^* = \frac{(B + Ah)\bar{I} + (1-A)h - C_{S1}}{h\bar{I} + d + D} ,$$

therefore

$$TRI_1 = \left(\frac{C}{B}\right) TRI_1^* . \tag{44}$$

Tables 1 through 11 in Appendix F give TRI_1^* for $h = h_{op}$. Then TRI_1 may be calculated by multiplying TRI_1^* by $R(Y-s) = C/B$ and T_M is determined by

$$T_M = - H_R(TRI_1) - f_x . \tag{45}$$

Equation (45) reveals that if $TRI_1^* > 0$, then $T_M < 0$. When A is close to one, and $C_{S1} \leq 100.0$, then TRI_1^* is usually positive and the criterion minimizes net loss for the year.

The nature of the expected net income equation in some neighborhood of h_{op} is of practical importance. For example, the value of h_{op} may be twenty-six minutes and because of scheduling difficulties it would be easier for the inspector if h_{op} were thirty minutes. In order to decide on deviating from h_{op} by four minutes the penalty of not being at maximum net income should be known. Tables 1 through 11 give the value of TRI_1^* for four selections of h: two less than h_{op}, and two greater than h_{op}. In many cases h may be as large as $(5)h_{op}$ and the change in TRI_1^*, from its value when $h = h_{op}$, is only 0.1 per cent. It is the variation in the expected net income, not TRI_1^* alone, which is important. If $B = 0.0001$, $C_{S1} = 500.0$, $\alpha_o = 1.0$, $\lambda = 0.01$, and $A = 0.9$, then (see Table 10) if h varies from h_{op} to $(5)h_{op}$, it is seen that TRI_1^* changes by only 0.12 per cent. However, the expected net income will be less than T_M by the potentially large amount

$$H_R\ (0.0012)(TRI_1^*(h = h_{op}))\ C/B =$$

$$H_R\ C\ (12.0)(4.056)\ \frac{\text{dollars}}{\text{year}}\ ,$$

when $h = 5\ h_{op} = 5(0.0623) = 0.3115$ hours.

In general, it is concluded (see Figure 6) that expected net income decreases rapidly (from its maximum) when $h < h_{op}$, and decreases relatively slowly (from its maximum) for $h > h_{op}$.

The TRI_2 Equation - Sequential Screening

Substituting the expression for $\overline{N}_o$, equation (32), into equation (36) reveals that h_{op} will be determined by the h coordinate at the minimum of the TRI_2 equation where

$$\mathrm{TRI}_2 = \left\{ \frac{C\bar{I} + R(Y-s)\bar{H}_b + S\rho(R(h+d)) - C_S}{h\bar{J} + d + D} \right\} . \qquad (46)$$

It is assumed that $h \notin J$ (see equation (31)), then equation (46), is differentiable with respect to h. After differentiation and simplification equation (46) becomes ($h \notin J$)

$$\psi(h;i) = \frac{(1-A)}{2^i}\alpha_o e^{-\lambda h} + \left\{ C_{S2} + \alpha_o - B + (1-A)\left[\frac{\alpha_o+h}{2^i} - \frac{\rho(R\{h+d\})}{R}\right]\right\} e^{\lambda h}$$

$$- \lambda h \left[C_{S2} + \alpha_o - \frac{(1-A)}{R} \cdot \rho\left(R\left\{h+d\right\}\right)\right] -$$

$$\left\{ \lambda\alpha_o B + \alpha_o + C_{S2} + (1-A)\left[\frac{2\alpha_o+h}{2^i} - \frac{\rho(R\{h+d\})}{R}\right]\right\} = 0 , \qquad (47)$$

where A, B, and α_o are defined in equations (40), (41), and (42) respectively, and

$$i = \left[\left[\frac{\log\left\{R(h+d)\right\}}{\log 2} \right]\right] ,$$

$$C_{S2} = \frac{1}{\lambda} + \frac{(V-Y)R\lambda^{-1} - (C_d \cdot D + R_C)}{R(Y-s)} - d > 0 . \qquad (48)$$

<u>Solution of the TRI_2 Equation.</u> The TRI_2 equation will not be solved in the general case but a method of obtaining a value for h_{op} will be discussed in this section.

The derivative of TRI_2 with respect to h does not exist at the points(*)

(*)The graph of ρ( ) as a function of h is a series of straight lines, with decreasing slopes, joined at the points (k,h_k), $(k+1,h_{k+1})$.

$$h \in J = \left\{ h_k = \frac{2^k}{R} - d \;\middle|\; k = 1,2,3,\ldots \right\} . \tag{49}$$

Equation (47) may have one or more positive zeros, or no positive zero because $\psi(h;i)$ is discontinuous when $h \in J$ and $\psi(h;i)$ has a negative jump as h crosses the points in the set J. It is difficult to say just how many positive zeros equation (47) has without knowledge of the values of the parameters. There are, however, a finite number of positive zeros because $\psi(0;0) < 0$ and $\psi(h;i) < 0$ for $h > H^*$, $H^* < +\infty$.

In any practical case $\psi(h;i) > 0$ for some $h > 0$, which implies TRI_2^* has a relative minimum for $h > 0$,(*) because

$$\lim_{h \to 0} TRI_2 = +\infty$$

and

$$\lim_{h \to \infty} TRI_2 = R(Y-s) .$$

It is noticed that equation (47) is equation (39) when $A = 1$. Therefore, h_{op} is the same for sequential or continuous screening when $A = 1$ (or $S = 0$).

The following procedure outlines a method that may be used to locate h_{op} when $A \neq 1$.

(*) First move the two terms that do not contain $e^{\lambda h}$ or $e^{-\lambda h}$ to the right side of equation (47). When $h = 0$ the right side is greater than the left side, or $\psi(0;0) < 0$. When h increases it is noticed that both sides first increase and then decrease and finally become negative. If $\psi < 0$ for all $h > 0$, then $h_{op} = +\infty$, and this case is completely unpractical. It is assumed that $(C_{S2} + \alpha_o - B) > 0$ is large enough so that $\psi > 0$ for some $h > 0$ and that TRI_2 is less than $R(Y-s)$ at at least one relative minimum point, otherwise $h_{op} = +\infty$.

(1) Enter the Tables, or refer to Figure 5, in Appendix F with $A = 1$ and calculate the optimum inspection interval. Call this value h^*_{op}.

(2) Calculate $\psi(h_k;k)$ for $k = 1,2,3,\ldots, \bar{k},\ldots, k_{max}$, where

$$\bar{k} = \left[\!\left[\frac{\log(R\left\{h^*_{op} + d\right\})}{\log 2}\right]\!\right] + 1 , \qquad (50)$$

and k_{max} is sufficiently large so that $\psi(h_{k_{max}};k_{max}) < 0$ and $\psi < 0$ for all $h > h_{k_{max}}$(*). Notice that $h_{\bar{k}} > h^*_{op}$.

(3) Define(**)

$$K = \left\{k \,\middle|\, k < k_{max},\ \psi(h_k;\ k) < 0 \text{ and } \psi(h_{k+1};\ k) > 0, \right. \qquad (51)$$

$$\left. \text{or } \psi(h_k;\ k) > 0 \text{ and } \psi(h_{k+1};\ k) < 0 \right\},$$

$$H = \left\{h \,\middle|\, k_o \in K,\ \psi(h;\ k_o) = 0, \right. \qquad (52)$$

$$\left. h_{k_o} < h < h_{k_o+1} \right\}.$$

Therefore, h_{op} is that $h \in H$ such that TRI_2 takes on its least value.

Probably the most efficient method would be to start with $k = \bar{k}$ and work downward through the numbers $k, k-1,\ldots,$ until $\psi(h_a;\ a-1) < 0$

(*) In many practical examples $h_{op} < h^*_{op}$, but there is the possibility that $h_{op} > h^*_{op}$ and the number k_{max} should be used.

(**) It is seen that (see equation (47))

$$\lim_{h \to h_j^+} \psi(h;\ i) = \psi(h_j;\ j), \text{ and}$$

$$\lim_{h \to h_j^-} \psi(h;\ i) = \psi(h_j;\ j-1) .$$

for some positive integer $a < \bar{k}$. Then $h_{op} > h_a$, and a similar scheme would be used for $k > \bar{k}$ to obtain an upper bound on h_{op}. This procedure was used for the TRC_2 equation (to be studied in a following section) and it was found that $h_{op} \in (h_{\bar{k}-1}, h_{\bar{k}})$ for the majority of cases studied.

The TRC Equations

The TRC_1 Equation - Continuous Screening

Substituting the expression for $\bar{N}_o$, equation (23), into equation (37) reveals that the criterion of maximizing net income results in locating the positive zeros of

$$\frac{d}{dh}\left\{C\bar{I} + R(Y-s)\bar{H}_b + S\left(R\left\{(h+d) - \bar{H}_b\right\} + 1\right)\right\} = 0. \qquad (53)$$

Using equation (6) of Appendix D for $\bar{H}_b$, then after differentiation and simplification equation (53) becomes

$$(1-A)e^{-\lambda h} + e^{\lambda h} - A\lambda h - (2 + \lambda B - A) = 0 , \qquad (54)$$

where A and B are defined in equations (40) and (41) respectively.

It is easy to see that there is a unique positive zero for equation (54) if $0 \leq A \leq 1$, a condition that holds in any practical case.(*) This unique positive root will be the h coordinate at the minimum of TRC_1, or h_{op}, since

$$\lim_{h \to 0} TRC_1 = +\infty ,$$

(*)First transfer all terms free of $e^{\lambda h}$ and $e^{-\lambda h}$ to the right side of equation (54). When $h = 0$ the right side is greater than the left side and as h increases both sides increase. The right side increases linearly in h, which implies equality holds for only one $h > 0$.

and

$$\lim_{h \to \infty} TRC_1 = +\infty .$$

Solution of the TRC_1 Equation. If $A = 0$ (or $Y-s = S$) then the direct solution of equation (54) is

$$h_{op} = \lambda^{-1} \ln\left[1 + \frac{\lambda B}{2} + \left\{\left(1 + \frac{\lambda B}{2}\right)^2 - 1\right\}^{0.5}\right].$$

Equation (54) was solved for its positive root for various values of the parameters A, B, and λ and Tables 12 through 19, and Figures 7 and 8, in Appendix G contain the results. Only positive values for A were used because a negative A, or $S > Y-s$, is of little practical importance.

The positive root of equation (54) was calculated by use of the Burroughs 220 Digital Computer at the Rich Electronic Computer Center at Georgia Institute of Technology. The results are accurate to all the decimal places displayed in the tables.

Variation in TRC_1 for Changes in h About h_{op}. Let TRC_1^* be defined as

$$TRC_1^* = (B + Ah)\overline{I} + (1-A)h ,$$

then

$$TRC_1 = \frac{C}{B}\left(TRC_1^* + d - \frac{A}{\lambda}\right) + S . \qquad (55)$$

Tables 12 through 19 in Appendix G also give the values of TRC_1^* at $h = h_{op}$. Then TRC_1 may be calculated from equation (55) and the minimum total variable cost, E(v), is determined by

$$E(v) = \frac{\lambda}{R}\,(TRC_1 + D \cdot C_d + R_C) + Y \,.$$

The shape of the expected net income equation in some neighborhood of h_{op} is of practical importance. Tables 12 through 19 in Appendix G also give the value of TRC_1^* for four selection of h: two less than h_{op}, and two greater than h_{op}. For example, if B = 0.0001, λ = 0.1 and A = 0.9, then TRC_1^* changes by 0.83 per cent when h increases from h_{op} to $(5)h_{op}$ (see Table 17). It follows that expected variable cost will be greater than the minimum E(v) by the amount

$$\frac{\lambda}{R} \cdot \frac{C}{B} \left\{0.0083\right\} (TRC_1^* \;(h = h_{op})) = (8.3)(9.047)\; C/R \;\frac{\text{dollars}}{\text{unit}} \,,$$

when $h = 5h_{op} = 5(0.0426) = 0.2130$ hours.

In general, it is concluded (see Figure 8) that expected variable cost per non-defective increases quickly (from its minimum) for $h < h_{op}$, and increases relatively slowly (from its minimum) for $h > h_{op}$.

The TRC_2 Equation - Sequential Screening

Substituting the expression for $\overline{N}_o$, equation (32), in equation (37) reveals that h_{op} will be determined by the h coordinate at the minimum of the TRC_2 equation, where

$$TRC_2 = C\overline{I} + R(Y-s)\overline{H}_b + S\rho \left(R \left\{h+d\right\}\right). \tag{56}$$

If it is assumed that $h \notin J$ (see equation (31)), then equation (56) is differentiable with respect to h. After differentiation and simplification equation (56) becomes $(h \notin J)$

$$\psi(h;\, i) = e^{\lambda h} - (\lambda h + \lambda B + 1) + \frac{2(1-A)}{2^i}\left\{\cosh(\lambda h) - 1\right\} = 0, \tag{57}$$

where

$$i = \left[\!\!\left[\frac{\log\left(R\left\{h+d\right\}\right)}{\log 2} \right]\!\!\right] ,$$

and B and A are defined by equations (41) and (40) respectively.

Solution of the TRC_2 Equation. The value of h_{op} will be located by a procedure similar to the one outlined for the TRI_2 equation in a previous section.

Equation (57) has at least one positive zero because $\psi(0;\ 0) = -\lambda B$ and $\lim_{h \to \infty} \psi(h;\ i) = +\infty$. The first positive zero will be a relative minimum because

$$\lim_{h \to 0} TRC_2 = +\infty$$

and

$$\lim_{h \to \infty} TRC_2 = +\infty .$$

Equation (57) may have one, two, or more positive zeros because $\psi(h;\ i)$ has a negative jump when h crosses the points in the set J. In all the numerical examples studied (see Appendix H) h_{op} was unique.

It is noticed that equation (57) is equation (54) when $A = 1$ (or $S = 0$). Hence, h_{op} is the same for sequential or continuous screening when $A = 1$.

The following procedure outlines a method for determining h_{op} when $A \neq 1$.

(1) Enter the Tables, or refer to Figure 7, in Appendix G with $A = 1$ and calculate the optimum inspection interval. Call this value h^*_{op}.

(2) The same format outlined for the TRI_2 equation applies here

except now it can be shown that

$$(h_{op})_{A<1} < (h_{op})_{A=1} .$$

Therefore, calculate $\psi(h_k; k-1)$ for $\overline{k}$, $\overline{k}-1$, $\overline{k}-2$, ..., k_o, $k_o \geq 1$, where

$$h_k = \frac{2^k}{R} - d ,$$

$$\overline{k} = \left[\left[\frac{\log(R\left\{h_{op}^* + d\right\})}{\log 2}\right]\right] + 1 ,$$

and $h_{\overline{k}} > h_{op}^*$, and

$$\psi(h_{k_o}; k_o - 1) < 0 ,$$

and it follows that $h_{op} \in (h_{k_o}, h_{\overline{k}})$. In the majority of numerical cases studied (see Appendix H) $h_{op} \in (h_{\overline{k}-1}, h_{\overline{k}})$.

(3) If there is more than one positive zero for $h \in (0, h_{\overline{k}})$, then h_{op} is that positive zero where TRC_2 takes on its least value.

The value of h_{op} was calculated for various selections of the parameters A, B, λ, and R, with $d = 0$. Tables 20 through 26, and Figures 9 and 10, in Appendix H contain the results.

The positive zero of equation (57) was located by the Burroughs 220 Digital Computer at the Rich Electronic Computer Center at Georgia Institute of Technology. The results are accurate to all the decimal places shown in the tables.

<u>Variation in TRC_2 for Changes in h about h_{op}.</u> Let TRC_2^* be defined as

$$TRC_2^* = (B + h)\overline{I} + \frac{(1-A)}{R} \cdot \rho\left(R\left\{h+d\right\}\right) ,$$

then

$$TRC_2 = \frac{C}{B}\left\{TRC_2^* + d - \frac{1}{\lambda}\right\}. \tag{58}$$

Tables 20 through 26 in Appendix H give the value of TRC_2^* ($d=0$) at $h = h_{op}$. The value of TRC_2 is obtained from equation (58) and the minimum total variable cost is determined by

$$E(v) = \frac{\lambda}{R}\left(TRC_2 + D \cdot C_d + R_C\right) + Y .$$

Tables 20 through 26 in Appendix H also give TRC_2^* for two selections of h less than h_{op}, and for two values of h greater than h_{op}. In general (see Figure 10) it is noticed that TRC_2^* (and hence $E(v)$) increases rapidly for $h < h_{op}$, and will increase relatively slowly for $h > h_{op}$.

Effect of Variation in Risk and Cost Factors on the Optimum Design

TRC_1 Equation: Minimizing Costs - Continuous Screening

A study of Figure 7 in Appendix G reveals that if $\lambda \leq 0.1$ or if $B \leq 1.0$ then the relation between $\log h_{op}$ and $\log B$ is a straight line for fixed λ and A. This relation can be expressed as

$$h_{op} \doteq K_{\lambda,A}\, B^m , \tag{59}$$

where

$$m = \text{the slope} = 0.500, \text{ and}$$

$$K_{\lambda,A} = h_{op} \quad \text{when } B = 1.0 .$$

Equation (59) can also be deduced in the following manner:

(1) expand $\overline{I}$ in an alternating series and approximate $\overline{I}$ by the first two terms ($\overline{I} \doteq 1/\lambda h + 1/2$), then

(2) use this approximation, and the corresponding approximations for $\overline{H}_b$ and $\overline{L}$, in the TRC_1 equation, differentiate and solve for h_{op}. This procedure produces

$$m = 1/2, \text{ and}$$

$$K_{\lambda,A} = \sqrt{\frac{2}{\lambda(2-A)}} \quad . \tag{60}$$

Hence

$$h_{op} \doteq \overline{h} = \sqrt{\frac{2B}{\lambda(2-A)}} = \sqrt{\frac{(\frac{1}{\lambda})\cdot 2C}{R(Y-s+S)}} \quad . \tag{61}$$

The value of $\overline{h}$ is within 2 per cent of h_{op} when $\lambda B < 0.1$.(*)

Figure 7 and equation (61) suggest the following general results ($\lambda B < 0.1$).

(1) h_{op} is directly proportional to the square root of the mean time to a failure (λ^{-1}), and the inspection cost (C).

(2) h_{op} is inversely proportional to the square root of the production rate (R) and the total cost of a defective ($Y - s + S$).

The above results hold in most practical cases because the cost ratio B and the failure rate λ will usually both be less than one.

(*)The approximation expressed in equation (61) grows rapidly worse when $\lambda h \gg 0.1$. A good approximation for h_{op} when $\lambda B > 0.1$, or $1 > \lambda h > 0.1$, is to use the first three terms in the series expansion of $\overline{I}$ for the approximation of $\overline{I}$. When $\overline{I} \doteq 1/\lambda h + 1/2 + \lambda h/12$, then $\overline{h}$ is the positive zero of a cubic equation, and a closed form solution for $\overline{h}$ is obtainable (see (1), (17), (21)).

TRI_1 Equation: Maximizing Income - Continuous Screening

Figure 5 in Appendix F shows that the relation between log h_{op} and log B is a straight line for fixed λ, A, C_{S1}, and α_o. This relation is expressed as

$$h_{op} \doteq K_{\lambda,A,C_{S1},\alpha_o} B^m , \tag{62}$$

where

$$m = 0.500, \text{ and}$$

$$K_{\lambda,A,C_{S1},\alpha_o} = h_{op} \text{ when } B = 1.0.$$

Equation (62) can also be deduced through the following procedure:

(1) expand $\bar{I}$ in an alternating series and approximate $\bar{I}$ by the first two terms,

(2) use the approximation for $\bar{I}$ in the TRI_1 equation, differentiate and solve for h_{op}.

This scheme gives $\bar{h}$, the approximation for h_{op}, as the positive zero of the following polynomial.

$$a_2 h^2 + a_1 h + a_o = 0 , \tag{63}$$

where

$$a_2 = C_{S1} - \frac{B}{2} + (2-A)\alpha_o + 2(1-A)\lambda^{-1} ,$$

$$a_1 = - 2B \lambda^{-1} ,$$

$$a_o = -2 B(\lambda\alpha_o + 1) \lambda^{-2} .$$

Substituting the expressions for C_{S1}, B, and A into the above equations for a_2, a_1, and a_o produces

$$b_2 h^2 + b_1 h + b_o = 0 , \tag{64}$$

where

$$b_2 = (V-Y)R\lambda^{-1} - (C_d D + R_C) - \frac{C}{2} +$$

$$(Y-s + S)\cdot R\cdot(\lambda^{-1} + D) + S(R\cdot d - 1) , \tag{65}$$

$$b_1 = 2C\cdot\lambda^{-1} , \tag{66}$$

$$b_o = 2C(\lambda\alpha_o + 1)\,\lambda^{-2} . \tag{67}$$

Solving equation (64) for $\bar{h}$ gives

$$h_{op} \doteq \bar{h} = \frac{C + \sqrt{C}\ \ (C + 2(1 + \lambda\alpha_o)b_2)^{\frac{1}{2}}}{\lambda b_2} . \tag{68}$$

The value of $\bar{h}$ is within 2 per cent of h_{op} if $B \leq 1$ and $C_{S1} \geq 100$. If $B = 1$ and $C_{S1} = 10$ then $\bar{h}$ can deviate from h_{op} by as much as 8 per cent.(*)

Figure 5 and equation (68) suggest the following general conclusions.

(1) When the sale price (V), the total cost of a defective (Y-s+S), R, or λ increase then h_{op} decreases.

(2) If the down time cost $(C_d\cdot D + R_C)$, inspection time (d), or C increase then h_{op} increases.

(*)Using the first three terms in the expansion of $\bar{I}$ then $\bar{h}$ is the positive zero of a fourth degree equation. This method gives a good approximation for h_{op} when $1 > \lambda h > 0.1$, or $C_{S1} < 100$ and $B = 1, 10$.

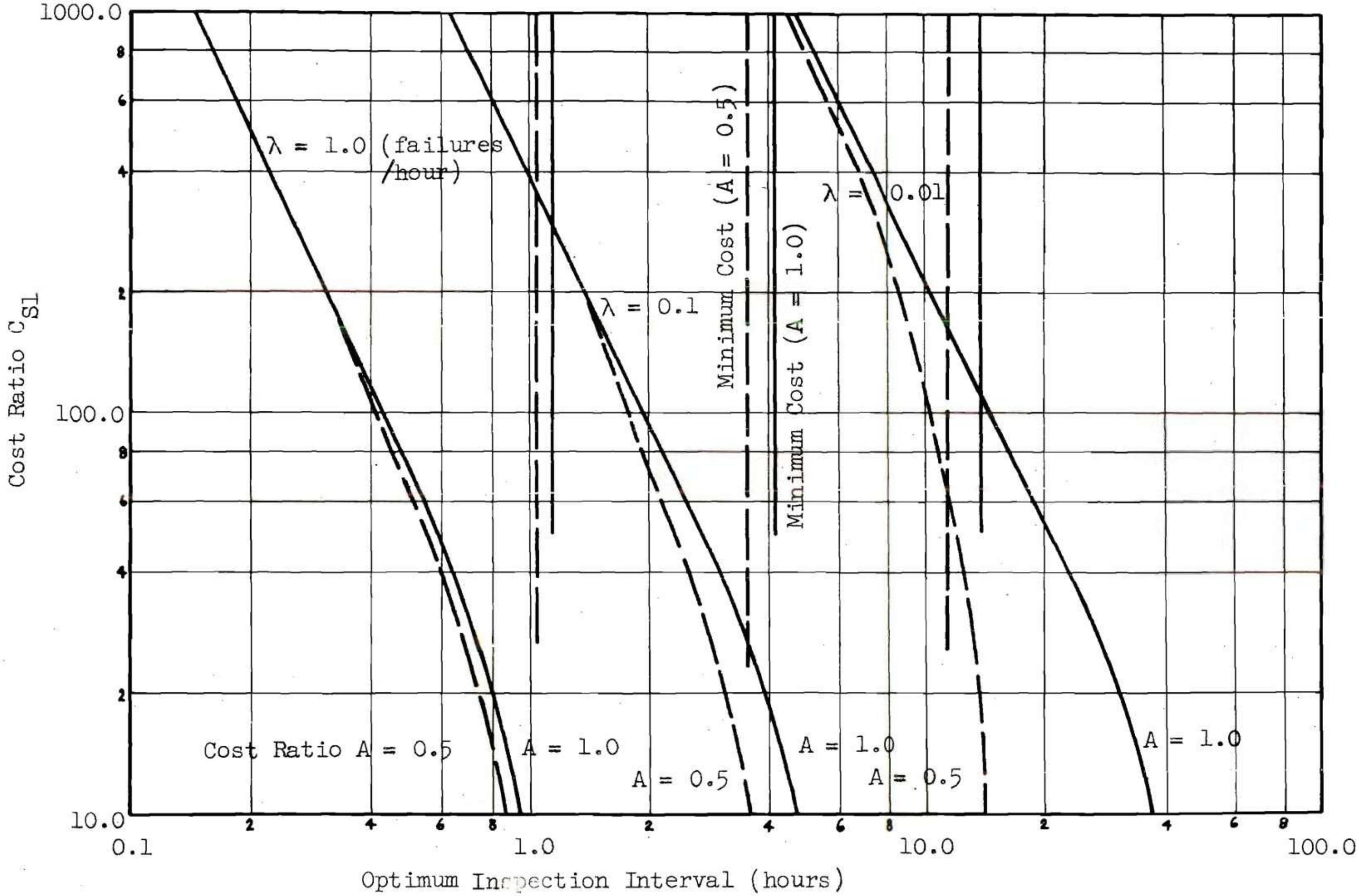

Figure 1. Relation Between Inspection Interval for Maximum Income and Inspection Interval for Minimum Cost - Continuous Screening and Cost Ratio B = 1.0 and α_o = 10.0 Hours.

(3) Variation in V (or C_{S1}) has a dominant effect on the optimum inspection interval. When V is large, or the margin for profit is great, then h_{op} should be small; when V is small then h_{op} should be large.

(4) The optimum inspection interval is approximately directly proportional to $2C$.

(5) Variation in the total cost of a defective has little effect on h_{op} if V is large, but changes in $S/(Y-s)$ cause h_{op} to vary by as much as 400 per cent when V (and hence C_{S1}), and λ are small (see Figure 1). From equation (68) it is deduced that as Y, or S increase then h_{op} decreases and when the salvage income (s) increases then h_{op} increases. Therefore, as the total cost of a defective grows large, h_{op} should be small.

(6) Variation in the total lag time ($\alpha_o = d + D$) from a defective inspection to the beginning of a new tool cycle, affects h_{op} more when V is large. When d, or D increase then C_{S1} decreases and α_o increases and therefore h_{op} increases.

(7) An increase in R will cause the cost ratio B to decrease and C_{S1} to increase, hence h_{op} will decrease.

(8) The effect on h_{op} of changes in λ^{-1} is greatest when λ and α_o are small. It is difficult to see the resultant change in h_{op} for variations in λ^{-1} because λ^{-1} is contained in C_{S1}. An analysis of the results displayed in Tables 1 through 11 reveals that if λ^{-1} increases then h_{op} also increases.

(9) If $TRI_1^* > 0$, which implies that the maximum net income is negative, then increases in α_o cause decreases in h_{op}.

Tables 1 through 11 suggest the following results, with respect to maximum income, T_M.

(1) When V, R, or λ^{-1} increase then T_M increases.

(2) If $(C_d D + R_C)$, C, d, or $(Y-s + S)$ decrease then T_M increases.

TRC_2 Equation: Minimizing Costs - Sequential Screening

Figure 9 in Appendix H shows that if $\lambda B \leq 1$ then the relation between log h_{op} and log B is a straight line. Therefore

$$h_{op} \doteq K_{\lambda,A,R,d} \; B^m , \tag{69}$$

where

$$m = 0.500, \text{ and}$$

$$K_{\lambda,A,R,d} = h_{op} \text{ when } B = 1.0.$$

Equation (69) is easily deduced by using the approximation(*) $\bar{I} \doteq 1/\lambda h + 1/2$, and it can be shown that

$$h_{op} \doteq \bar{h} = \left[\lambda^{-1} \, 2^i B \Big/ (2^{i-1} + 1 - A) \right]^{\frac{1}{2}} , \tag{70}$$

where

$$i = \left[\left[\frac{\log \left\{ R(\bar{h}+d) \right\}}{\log 2} \right]\right] , \tag{71}$$

if $\bar{h} \notin J$, and the set J is defined by equation (31).

Hence

(*) See either of the two previous sections.

$$\bar{h} = \left[\lambda^{-1}2^{i}\ C \Big/ \left\{R(Y-s)2^{i-1} + RS\right\}\right]^{\frac{1}{2}} \tag{72}$$

The value of $\bar{h}$ is within 2 per cent of h_{op} if $B<1$, or if $B=1$ and $\lambda<0.1$. When $B = 100$, then $\bar{h}$ can deviate from h_{op} by as much as 250 per cent when $\lambda = 1.0$(*). It is noticed that a solution to equation (72) would be by trial and error. To calculate $\bar{h}$ one would first test the points

$$h_k = (2^k / R) - d,\ k = 1,2,\ldots$$

and then search for $\bar{h}$ in the proper interval (h_k, h_{k+1}), until equation (72) is satisfied.

Figure 9 and equation (72) suggest the following conclusions $(\lambda \cdot B < 1)$.

(1) h_{op} is directly proportional to the square root of C and λ^{-1}.

(2) h_{op} is inversely proportional to the square root of R and $(Y - s + S)$.(**)

Comparison: Continuous and Sequential Screening - U Fixed

Let $h_{op} = h_C$ when continuous screening is used and $h_{op} = h_S$ when sequential search is employed. It can be shown that

$$(h_{op})_{A=1} > (h_S)_{A<1} > (h_C)_{A<1}\ .$$

When $A = 1$, then $h_S = h_C$ because if $A = 1$ then the unit screening cost is zero.

(*) The approximation for h_{op} can be made better by using more terms to approximate $\bar{I}$. See the discussion in either of the two previous sections.

(**) This is not strictly correct for R because i is a function of R.

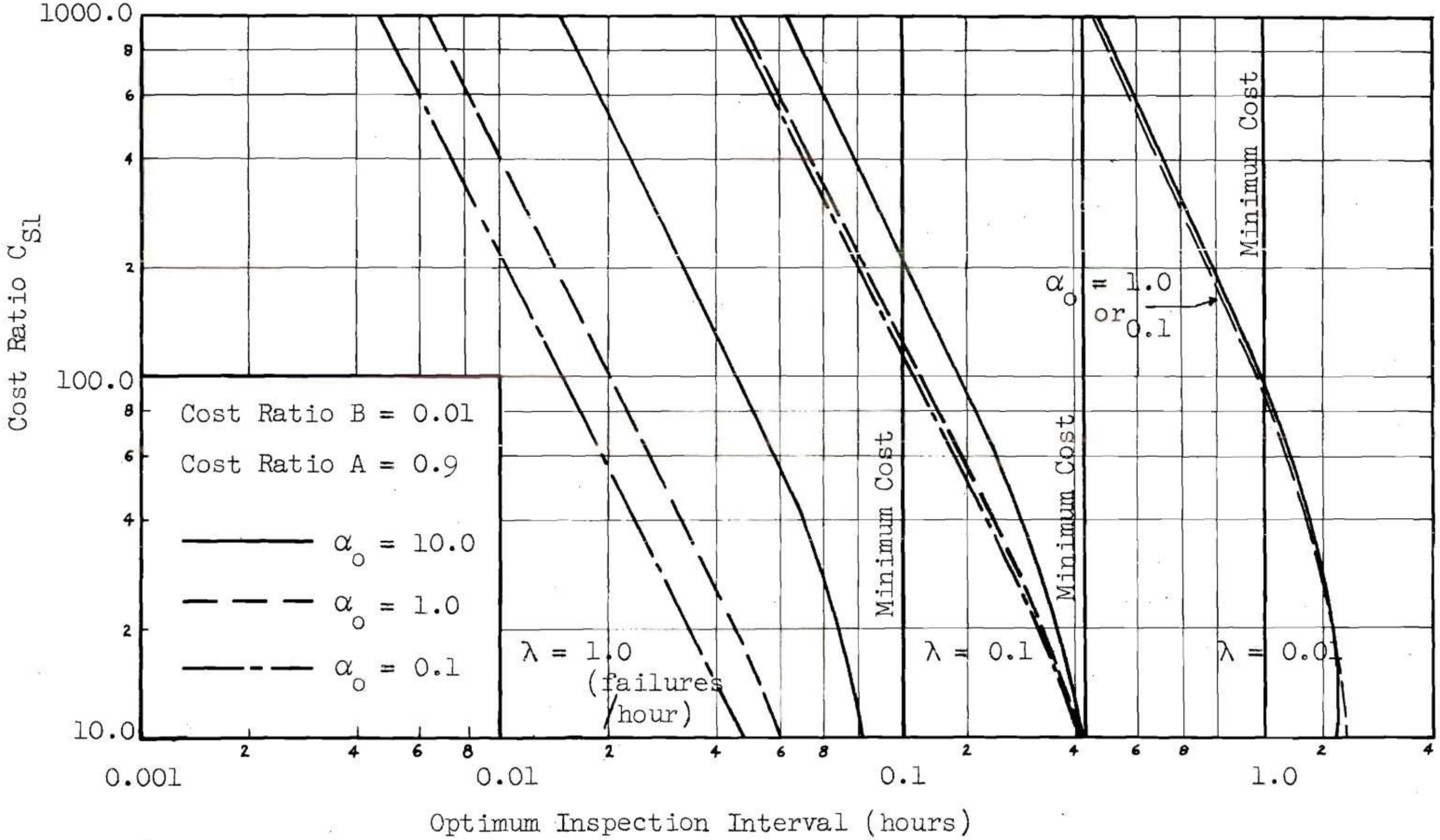

Figure 2. Relation Between Inspection Interval for Maximum Income and Inspection Interval for Minimum Cost - Continuous Screening and Cost Ratios A and B Constant.

Let $\Delta = h_S - h_C$.

Figures 1, 2, and 3 and Tables 12 through 26 suggest the following results.

(1) Δ is largest when C, or λ is small, or (Y-s) and S are large.

(2) Variation in the ratio $S/(Y-s)$ effects h_C more than h_S.

(3) Δ/h_C is close to 33 per cent when A = 0.25, λ = 0.01, and $B < 1$; Δ/h_C is usually 20 per cent when $A \leq 0.5$ and $\lambda > 0.01$; Δ/h_C is approximately 2 per cent when A = 0.9, i.e. when S is close to zero.

(4) Variation in the cost ratio B effects h_C and h_S in the same manner.

It can also be shown that

$$E(v)_C \;>\; E(v)_S$$

because

$$TRC_1 > TRC_2 \;,$$

or

$$(TRC_1^* - TRC_2^*) + (1-A)\;(\frac{1}{R} + \frac{1}{\lambda}) \;>\; 0. \tag{73}$$

The validity of equation (73) comes from the values of TRC_1^* and TRC_2^* in the Tables in Appendix G and H.

The "saving," $S_v = E(v)_C - E(v)_S$, obtained by changing from continuous to sequential screening is

$$S_v = \frac{\lambda}{R} \cdot \frac{C}{B} \left\{ (TRC_1^* - TRC_2^*) + (1-A)\;(\frac{1}{R} + \frac{1}{\lambda}) \right\} . \tag{74}$$

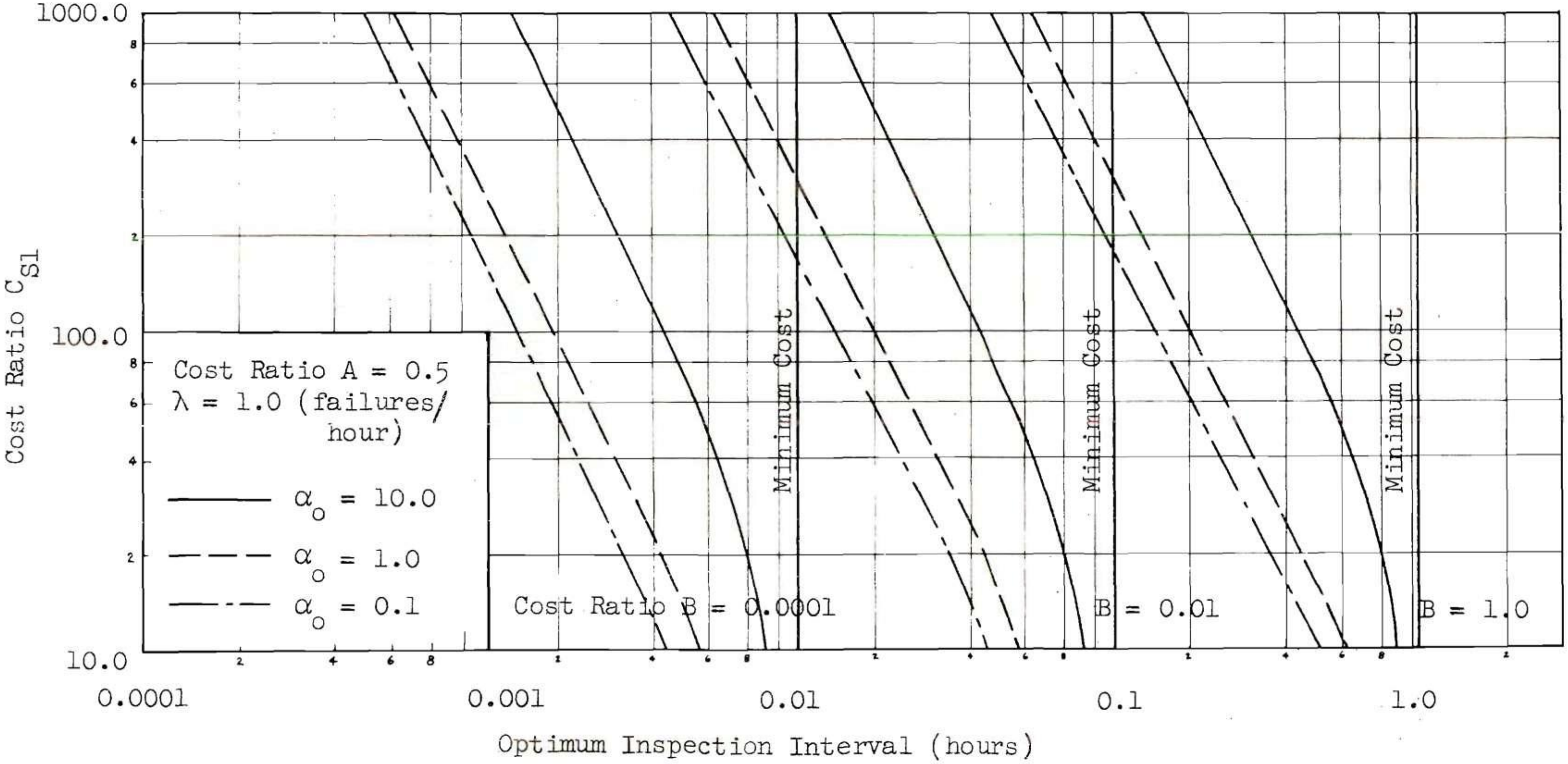

Figure 3. Relation Between Inspection Interval for Maximum Income and Inspection Interval for Minimum Cost - Continuous Screening and Cost Ratio A and Failure Rate λ Constant.

The following results can be gleaned from a study of the Tables in Appendix G and H.

(1) S_v increases when the unit screening cost, or the ratio $S/(Y-s)$ increases.

(2) S_v increases if λ decreases.

Comparison: Maximizing Income and Minimizing Cost - Continuous Screening

Let $h_v = h_{op}$ when the criterion is to maximize income, and $h_f = h_{op}$ when the criterion is to minimize costs, and $\Delta = h_f - h_v$.

Figures 1, 2, and 3 and Tables 1 through 19 reveal the following results.(*)

(1) Δ is usually positive.

(2) Δ increases when d, $(C_d \cdot D + R_C)$, or λ decrease.

(3) The sale price has the greatest effect on Δ. When V (or C_{S1}) increases then Δ increases.

(4) Δ is relatively unaffected by variation in C, (Y-s), and S.

(5) If R increases, then C_{S1} and Δ increase.

Conclusions

The optimum inspection interval is determined by the production rate (R), inspection cost (C), mean time to a failure (λ^{-1}), and the total cost of a defective (Y - s + S) when the criterion is to minimize costs. If the criterion is to maximize net income, h_{op} is determined by the above variables and by the sale price (V), downtime costs $(C_d \cdot D + R_C)$, and inspection time (d).

(*) Recall that h_f is fixed for all changes in α_o or C_{S1}.

The optimum inspection interval decreases when $(Y - s + S)$, R, or λ increase, or when C decreases.(*)

When the criterion is to maximize net income,(*) h_{op} is largely a function of V. If V increases then h_{op} decreases so that more non-defectives are produced to be sold at the higher sale price. The amount h_{op} decreases, from the value of h_{op} when the total number of units to be produced for the year is fixed, increases as the number of non-defectives produced per tool cycle (R/λ), or V increase. In most practical cases $(C_{S1} \geq 100)$ the value of h_{op}, when net income is maximized, will be less than the value of h_{op} when costs are minimized.

When the criterion is to minimize costs, the expected variable cost $(E(v))$ is less when sequential search is used if the screening activity does not damage the product. The reduction in $E(v)$, when sequential search is used in place of continuous screening increases when the unit screening cost (S), $\frac{1}{\lambda}$, or Rh increase. The value of h_{op}, when sequential search is employed, is larger than the value of h_{op} for continuous screening.

The Model fails when it is assumed that U is fixed for the year and $B >> 1.0$. When B grows very large, s will be close to Y, and h_{op} will be large. If s is close to the variable cost per unit (Y) then it becomes less costly to produce defectives. When this happens h_{op} will become large, more defectives will be produced, and the assumption that

---

(*)Not all of the results stated here necessarily apply to the case when sequential search is used and the criterion is to maximize income, because this case was not evaluated numerically.

the total non-defectives (U) made for the year is constant becomes invalid. Therefore, if s is close to Y, U should be a variable and the criterion of maximizing net income should be used. It is noticed that as s grows large, then (Y-s) becomes small and the cost ratio A is negative. The model is valid for $A < 0$, but this case was not investigated.

CHAPTER III

MODEL II

Introduction

In this chapter Model I is extended to include wearout failure. When wearout is present the conditional probability of the process surviving (not failing) to the time $x + t$ $(t > 0)$, if the process has not failed prior to the time x, decreases with the increasing age, x, of the machine tool. For this reason it is economical to introduce preventive maintenance.

In Model II the process will be controlled by an integrated quality control and preventive maintenance plan. It is assumed that the product fraction defective changes, as a result of a failure, from zero per cent to one hundred per cent during an infinitesimal period of time. The criterion developed in Model I is applied but no solutions are given. Only the case, where the inspection and screening activities do not damage the product in any way, will be studied.(*)

Infant mortality will not be considered in this model. Lloyd and Lipow, in (18) and Bazovsky, in (30), discuss how this problem may be incorporated into a failure model.

(*)If one wishes to study the case when screening damages the product, a similar analysis as that of Appendix E for Model I should be conducted for this model.

Examples(*)

(1) The missile example of Model I may also fail as a result of wearout. When a missile is standing in its silo ready to be fired there are a number of components in operation, for example the gyroscope, and wearout analysis is applicable.

(2) The automatic cutting tool or punch press example of Model I may also fail as a result of wearout. The cutting tool may become worn, or the punch press die may crack from metal fatigue. When wearout is significant this model should be used.

The Problem

A general type of process is investigated in which the production rate is a constant R discrete units per process running hour. When a failure occurs production immediately shifts from zero per cent defective to one hundred per cent defective and the process will continuously produce defectives until the failure is discovered and the machine tool is fixed.

The process may fail as a result of chance or wearout causes. Let there be n machine components whose failure results in defective production. It is assumed that the various causes of machine failure occur independently and component failures are mutually independent. The arrivals of process failure, as a result of chance causes, are considered to be at an average rate of λ occurrences per failure free process running

(*) A system with wearout components will progress to a state where all failures occur randomly in time when there is no preventive maintenance (see (30)). Model I applies here if one waits for the system to get "old."

hour. The arrivals of process wearout failure are at an average rate of λ_w occurrences per failure free process running hour.

The quality control procedure is to inspect the process every h process running hours and if the observed piece is non-defective let the process continue operating. When the piece is defective production is immediately stopped, past production is screened for defectives and the machine tool is returned to its non-failed state.

Let t = the process running hours measured from the beginning of the tool cycle. The preventive maintenance plan is to automatically stop production and change all wearout components when $t = R_o$. A new machine component may be a reconditioned tool (for example a resharpened cutting tool) or a previously unused tool. A reconditioned tool is assumed to be a statistically new tool.

A new variable K is now defined so that $Kh = R_o$. In theory, K could merely be restricted to be not less than one; however, our attention will be restricted to positive integer values of K only.(*)

The problem is to first select a maintenance plan and a process failure density function, and then apply the criterion defined by equation (2) of Chapter II, to obtain the optimum operating conditions.

Further Restrictions

Let there be n_r components that fail as a result of chance causes and n_w components fail as a result of wearout. Pieruschka, in (13),

---

(*)If K is continuous then the events that divide the criterion function into four parts, equation (5), need to be redefined and the expression for $E(I|\ell)$, $E(L|\ell)$, $E(H_b|\ell)$, and $E(N_o|\ell)$ will all change (see Appendix K). The additional complexity of the criterion function usually does not justify a continuous K.

shows that the distribution of time between failures for an assembly of n_r chance failure components is the negative exponential distribution with mean equal to the sum of the means of each component time to failure distribution.(*)

The distribution of the waiting time to the first process wearout failure is assumed to be $\Gamma(t)$. It is seen that the distribution of the time between the s and (s + 1)th process wearout failure is also $\Gamma(t)$ only if all wearout components are changed together so that the process is restored to zero age after any wearout failure.

The maintenance procedure to be used in Model II is:

(1) when a component, which fails only as a result of chance causes, fails prior to the time (K - 1)h, repair this component and restore the process to production,

(2) if a component, which fails only as a result of chance causes, fails in the Kth inspection interval, repair this component and change all wearout components,

(3) if a wearout component fails at any time, change all wearout components,

(4) if the process does not fail before the time (K-1)h, stop the process at R_o = Kh and change all wearout components.

Preventive maintenance only applies to components subject to wear-

---

(*)This is also studied and developed in Appendix A.

out because the failure rate of chance failure components is constant regardless of their age.

This maintenance plan is applicable to systems that require a high degree of reliability and/or when it is economical to completely overhaul the process whenever it fails from wearout.

Another maintenance plan would be to change all wearout components only at the preventive maintenance shutdown. The reader is referred to (13) for the general process failure distribution formulas for this case.

Obviously, there are an infinite number of possible maintenance plans. The question of what maintenance scheme is best, or optimum, will not be studied here.

Process Failure Density Function

The failure density function for a process that fails as a result of chance causes is assumed to be the exponential density function, defined by equation (1) of Chapter II. A survey of the literature on wearout models shows that there are only four models of general importance. Each wearout model is discussed in Appendix I. The gamma distribution is selected to describe the phenomena of wearout for Model II because of a derivation of a wearout model presented by Lloyd and Lipow (18). A condensation of this argument is in Appendix I. The "addition," by equation (3) of Appendix A, of the gamma and exponential densities gives the process failure density function for chance and wearout failures.[*] The method

---

[*] This "addition" gives the distribution of the waiting time to the first process failure. This distribution is preserved for the waiting time between any process failure because:

(i) all wearout components are changed together, and

(ii) chance failures are independent of the time since the last process failure.

of estimating the parameters of this density function is discussed in Appendix J.

Let $\gamma(t)$ be the gamma density function, then

$$\gamma(t) = \begin{cases} \dfrac{\mu^N t^{N-1} e^{-\mu t}}{\Gamma(N)} & t \geq 0 \\ 0 & t < 0 \end{cases} \tag{1}$$

with $\mu > 0$ and $N > 0$.

If $g(t)$ is the process failure density function, then (see Appendix I),

$$g(t) = \begin{cases} e^{-\lambda t}\gamma(t) + \lambda e^{-\lambda t} \left\{ \int_t^\infty \gamma(X)dX \right\}, & t \geq 0 \\ 0 & t < 0 \;. \end{cases} \tag{2}$$

If $G(t)$ is the process failure distribution function, then

$$G(t) = \int_0^t g(X)\, dX = 1 - e^{-\lambda t}\left\{\int_t^\infty \gamma(X)\, dX\right\} . \tag{3}$$

Terminology

In addition to all the terms defined in Chapter II for Model I the following terms are introduced for Model II.

Let R_o = the maximum process running hours, without a wearout failure, until an automatic preventive maintenance shutdown,

K = R_o/h, and it is assumed to be an integer,

$\gamma(t)$ = the wearout failure density function, defined by equation (1),

$g(t)$ = the process failure density function, defined by equation (2),

$G(t) = \int_0^t g(X)dX$, and is defined by equation (3),

λ_w = the mean number of process failures, as a result of wearout, per failure free process running hour,

$\frac{N}{\mu} = \frac{1}{\lambda_w}$ = the mean failure free process running hours to a wearout failure (N is assumed to be an integer and this number is the mean of $\gamma(t)$),

λ = the mean number of process failures, as a result of chance causes, per failure free process running hour,

n_r = the number of process components that fail only as a result of chance causes,

n_w = the number of process components that fail only as a result of wearout,

n_t = the number of process components whose failure results in defective production, and $n_t = n_r + n_w$,

λ_i = the mean number of chance failures per failure free process running hour for the i^{th} (chance failure) component; $i = 1, 2, \ldots, n_r$, and note $\lambda = \sum_{i=1}^{n_r} \lambda_i$,

s_i = the salvage income obtained from a defective produced because the ith chance failure component failed; $i = 1,2,\ldots,n_r$,

s_w = the salvage income obtained from a defective produced because of a wearout failure,

D_i = downtime for retooling the ith chance failure component,

D_w = downtime for changing all wearout components,

R_{c_i} = retooling cost for the ith chance failure component,

R_{c_w} = retooling cost when all wearout components are changed,

S_i = unit screening cost, when the ith chance failure component failed,

S_w = unit screening cost, when a wearout failure occurred, and

$P(E)$ = means the probability of event E.

For this model "tool cycle" means the process running hours to when a failure is discovered or to Kh, whichever occurs first, plus the downtime for overhaul or retooling.

All the symbols used in this chapter are presented in the List of Symbols in Appendix L for easy reference.

Development of the Criterion Function

The criterion to be used for selecting the inspection interval h and the time to an automatic preventive maintenance shutdown Kh, will be the same criterion used in Model I. Define the number T_M to be

$$T_M = \underset{\substack{h \geq 0 \\ K=1,2,\ldots}}{\text{Maximum}} \left[E \left\{ \frac{\text{net income}}{\text{year}} \right\} \right] . \qquad (4)$$

Assuming such a number T_M exists, let h_{op} be that value of $h \geq 0$, and K_{op} be the $K = 1,2,\ldots\ldots$, that produce the number T_M. Equation (4) will be the criterion used to determine a value for h and K.

The net income expression will be divided into four sections so that the policy of having a separate maintenance plan for wearout and chance failure components may be more readily expressed in terms of the variables h and K.(*) Consider(**)

$$E\left\{\frac{\text{net income}}{\text{year}} = I_N\right\} = \sum_{\ell=1}^{4} P(\ell)E(I_N|\ell), \tag{5}$$

where

$\ell = 1$ is the event that the process failed prior to $(K-1)h$ and the failure was the result of chance causes,

$\ell = 2$ is the event that the process failed in the Kth inspection interval and the failure was the result of chance causes,

$\ell = 3$ is the event that the process failed prior to Kh and the failure was the result of wearout, and

$\ell = 4$ is the event that the process did not fail by the time Kh.

When $\ell = 1$ only the failed component is changed. If $\ell = 2$ the failed component is changed and all wearout components are also changed. All the wearout components are changed for $\ell = 3$ and $\ell = 4$.

The following assumptions are made:

(1) the process is stopped only when a failure has been discovered or at the time R_o, whichever occurs first,

(2) the inspector inspects the last unit produced,

(3) only one component may fail in an inspection interval,(***)

(*) Otherwise, one must consider the expected number of chance failures prior to a process overhaul.

(**) By partitioning the event $\ell = 1$ into two disjoint events, this procedure may be extended to cover the case where some components fail as a result of both wearout and chance causes.

(***) Or alternately: the probability of two or more components failing in an inspection interval is small.

(4) the process is immediately stopped at the time R_o and then production is inspected,

(5) no defectives are sold, or the screening procedure eliminates all defectives.

Only sequential screening is considered, because on the average sequential screening screens less units per process failure than continuous screening.(*)

It is seen that

$$P(\ell=1) = \int_o^{(K-1)h} \lambda e^{-\lambda t} \int_t^{\infty} \gamma(X)dXdt, \tag{6}$$

$$P(\ell=2) = \int_{(K-1)h}^{Kh} \lambda e^{-\lambda t} \int_t^{\infty} \gamma(X)dXdt, \tag{7}$$

$$P(\ell=3) = \int_o^{Kh} \gamma(t)e^{-\lambda t}dt, \tag{8}$$

$$P(\ell=4) = \int_{Kh}^{\infty} g(t)dt. \tag{9}$$

Since (see equation (2))

$$g(t) = \lambda e^{-\lambda t} \int_t^{\infty} \gamma(X)dX + \gamma(t)\, e^{-\lambda t} \quad ,$$

then

$$\sum_{\ell=1}^{4} P(\ell) = 1 .$$

(*) See the footnote on page 24.

Now, from equation (3) of Chapter II

$$E(I_N) = VE(U) - E(Uv) - f_x \; .$$

Therefore, for $\ell = 1,2,3,4$

$$E(I_N \mid \ell) = VE(I_N \mid \ell) - E(Uv \mid \ell) - f_x \; ,$$

and equation (5) becomes

$$E(I_N) = \sum_{\ell=1}^{4} P(\ell) \left\{ VE(U \mid \ell) - E(Uv \mid \ell) \right\} - f_x \; . \tag{10}$$

Using equation (17) of Chapter II it follows that(*)

$$E(Uv \mid \ell) \doteq \frac{H_R E(M \mid \ell)}{E((L+D) \mid \ell)} \; ,$$

for $\ell=1,2,3,4$.

Using equation (11) of Chapter II, it is seen that

$$E(M \mid \ell) = CE(I \mid \ell) - RE(sH_b \mid \ell) + E(SN_o \mid \ell) + E(R_C \mid \ell) +$$

$$C_d E(D \mid \ell) + YRE(L \mid \ell), \tag{11}$$

and using equation (9) of Chapter II

$$E(\left\{L+D\right\} \mid \ell) = hE(I \mid \ell) + d + E(D \mid \ell) \; . \tag{12}$$

It is also seen, referring to equations (6) and (8) of Chapter II, that(*)

(*)See the footnote on page 13.

$$E(U|\ell) \doteq \frac{H_R RE(T|\ell)}{E(\,(L+D)\,|\ell)} , \tag{13}$$

for $\ell = 1,2,3,4$.

The expressions for $E(I_N|\ell)$ and $P(\ell)$, for $\ell = 1,2,3,4$, are derived in Appendix K.

The Criterion Function - U Variable

Substituting equations (11), (19), (30) and (38) of Appendix K into equation (5) it is seen that h_{op} and K_{op} are determined from the number D_M, where

$$D_M \doteq \underset{\substack{h \geq 0 \\ K=1,2,\ldots}}{\text{Max}} \left\{ \sum_{\ell=1}^{4} P(\ell) \cdot \left[\frac{(V - s_\ell)RE(T|\ell) - \left\{ \begin{array}{l} (C+Rh\left[Y-s_\ell\right])E(I|\ell) + \\ (R_C)_\ell + C_d D_\ell + R(Y-s_\ell)d_\ell + \\ S_\ell \rho(R\,(h+d_\ell)\,) \end{array} \right\}}{hE(I|\ell) + d_\ell + D_\ell} \right] \right\} , \tag{14}$$

since

$$D_M = (T_M + f_x)/H_R \,.$$

And

$$D_\ell = \begin{cases} \overline{D} = \sum_{i=1}^{n_r} D_i \lambda_i / \lambda & \text{if } \ell = 1 \\ D_w & \text{if } \ell = 2,3,4, \end{cases}$$

$$d_\ell = \begin{cases} d & \text{if } \ell = 1,3 \\ 0 & \text{if } \ell = 2,4, \end{cases}$$

$$S_\ell = \begin{cases} \overline{S} = \sum_{i=1}^{n_r} S_i\lambda_i/\lambda & \text{if } \ell = 1,2 \\ S_w & \text{if } \ell = 3 \\ 0 & \text{if } \ell = 4, \end{cases}$$

$$s_\ell = \begin{cases} \overline{s} = \sum_{i=1}^{n_r} \lambda_i s_i/\lambda & \text{if } \ell = 1,2 \\ s_w & \text{if } \ell = 3 \\ 0 & \text{if } \ell = 4, \end{cases}$$

$$(R_C)_\ell = \begin{cases} \overline{R}_C = \sum_{i=1}^{n_r} R_{C_i} \cdot \lambda_i/\lambda & \text{if } \ell = 1 \\ \overline{R}_C + R_{C_w} & \text{if } \ell = 2 \\ R_{C_w} & \text{if } \ell = 3,4, \end{cases}$$

and expressions for $P(\ell)$, $E(T|\ell)$, and $E(I|\ell)$ are derived in Appendix K. The expressions $P(\ell)$, $E(T|\ell)$, and $E(I|\ell)$ are functions of K and h, while $\rho(\ )$ is a function of h only.

The Criterion Function - U Constant

In Chapter II it was shown that in this case the criterion function reduced to minimizing the expected total variable cost (per non-defective) expression. Therefore, h_{op} and K_{op} are determined from the number G_M, where

$$G_M = \underset{\substack{h > 0 \\ K=1,2,\ldots}}{\text{Min}} E(v) = \underset{\substack{h > 0 \\ K=1,2,\ldots}}{\text{Min}} \sum_{\ell=1}^{4} P(\ell)E(v|\ell). \tag{15}$$

Using equation (14) of Chapter II it is seen that[(*)]

$$E(v|\ell) \doteq \frac{E(M|\ell)}{RE(T|\ell)} \quad ,$$

for $\ell = 1,2,3,4$, and $E(M|\ell)$ is expanded in equation (11).

It can be shown, consult Appendix K and the derivation of $E(v)$ in Model I, that

$$G_M \doteq \underset{\substack{h > 0 \\ K=1,2,\ldots}}{\text{Min}} \left\{ \sum_{\ell=1}^{4} P(\ell) \left[\frac{\left\{C+Rh(Y-s_\ell)\right\} E(I|\ell)+R(Y-s_\ell)d_\ell+(R_C)_\ell + C_d D_\ell + S_\ell \rho(R(h+d_\ell))}{RE(T|\ell)} \right] \right\} + s, \tag{16}$$

where s_ℓ, d_ℓ, D_ℓ, $(R_C)_\ell$, and S_ℓ are all defined in the previous section.

Conclusions

If all the process components fail as a result of wearout and $\bar{s} \doteq s_W$, $\bar{R}_C \doteq (R_C)_W$, $\bar{S} \doteq S_W$, and $\bar{D} \doteq D_W$ then any component failure affects the process in the same way. In this case there is no need to use conditional expected values in the criterion function and the procedure is parallel to that of Model I. The only difference is that the failure density function, now $g(t)$, is always truncated at the preventive maintenance shutdown. The criterion function is not developed for this case.

(*) See the footnote on page 13.

When there is at least one component that fails only as a result of chance causes the procedure of this model may be used. In order to find h_{op} and K_{op} for a particular set of parameters one may differentiate the expected net income equation with respect to h for fixed K. When $K = 1$, $h_{op}^{(1)}$ would be determined from the zeros of the differentiated equation, and this procedure could be continued for $K = 2, 3, \ldots, K_{max}$. The number K_{max} is sufficiently large so that it is seen that D_M, equation (14), will decrease for larger values of K. A similar procedure might be used when U is constant.

The resulting complexity of the criterion function, equation (14) or (16), will usually prohibit using differentiation as a tool in locating h_{op} and K_{op}. Probably the most expedient method would be to employ simulation, together with "experimental design" procedures to minimize the number of simulation runs required, to locate h_{op} and K_{op}.

CHAPTER IV

GENERAL MODEL

Introduction

In this chapter Model II is extended to the case where the product quality characteristic is distributed according to the probability law $X(t; \xi)$ (see Figure 4) where t is the process running hours measured from the beginning of the tool cycle, and ξ is the measure of the product quality characteristic. With this model it is necessary to have samples of more than one unit at each inspection, since the process quality is now a stochastic variable, rather than a deterministic variable as assumed for Models I and II.

The selection of a "quality control plan" and the application of the criterion of Model II are discussed, but no solutions are given. In this chapter a "quality control plan" will mean a typical control chart (i.e. an $\bar{X}$-chart or p-chart, etc.) used to detect changes in product quality.

Examples

(1) Let

t^* = the time to a process failure, or Kh, whichever occurs first, then set

$$\xi = \left\{ \begin{array}{ll} 1 & \text{with probability } p_1 \\ 0 & \text{with probability } 1 - p_1 \end{array} \right\} \text{for } t \leq t^*,$$

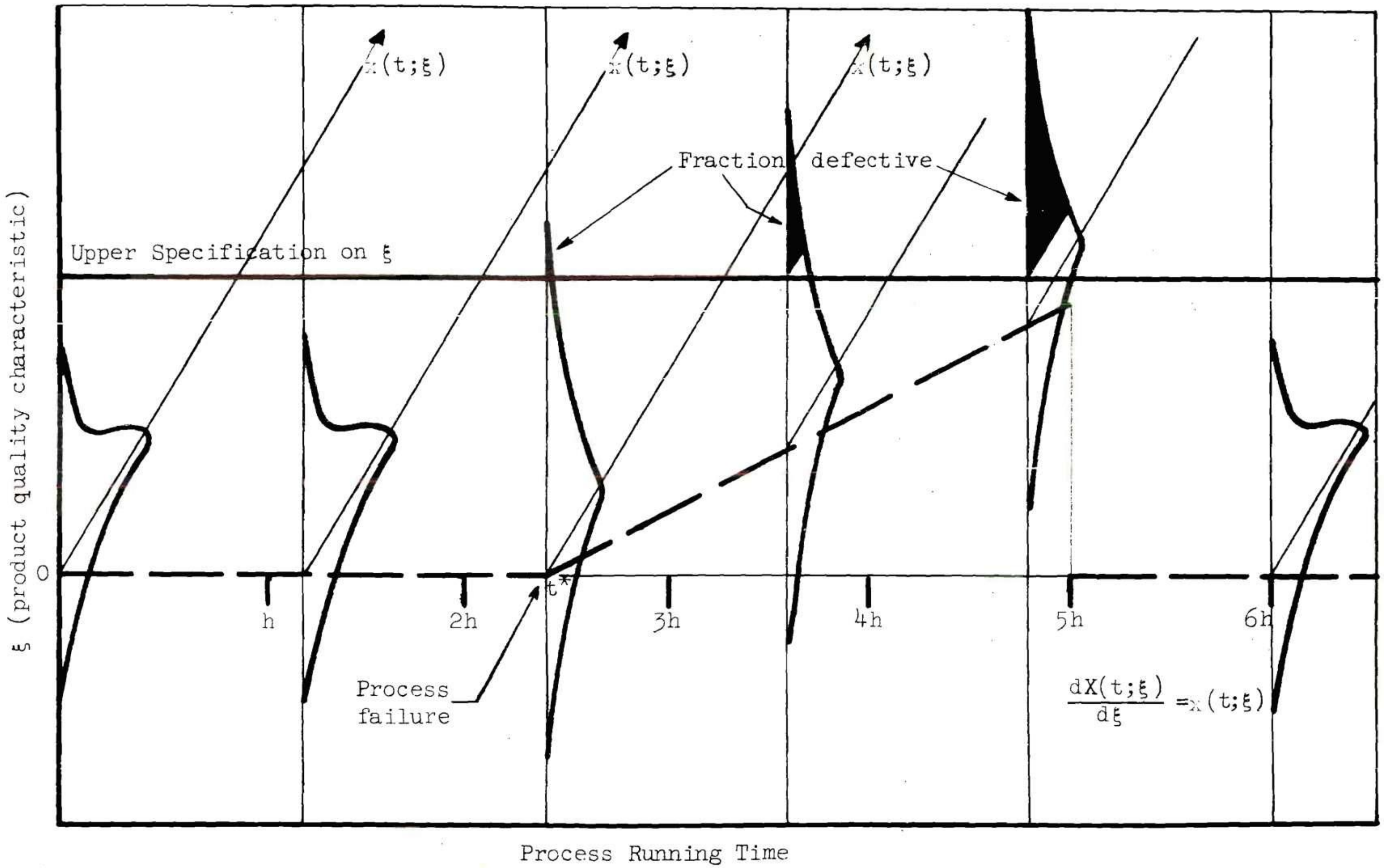

Figure 4. Product Quality Distribution and Process Failure.

$$\xi = \begin{cases} 1 & \text{with probability } p_2 \\ 0 & \text{with probability } 1 - p_2 \end{cases} \text{ for } t > t^*,$$

and assume $0 \leq p_1 < p_2 \leq 1$. Models I and II are studies of the situation of having $p_1 = 0$ and $p_2 = 1$. The numbers p_1 and p_2 may be thought of as the process fraction defective before and after a process failure respectively, or the "in-control" and "out-of-control" quality levels.

(2) When the quality characteristic exists on a continuous scale let

$$\xi = \begin{cases} \mu(t) + w_1 + \epsilon & \text{for } t \leq t^* \\ \mu(t) + w_2 + \epsilon & \text{for } t > t^*, \text{ and} \end{cases}$$

$\mu(t)$ = some known function of time (due perhaps to variation in material, temperature, etc.) with the special case of $\mu(t) = \mu$ for all t,

w_1 = a random variable, statistically independent of t and distributed normally with zero mean and variance $\theta_1^2\sigma^2$, with $\theta_1 > 0$,

w_2 = a random variable, statistically independent of t and distributed normally with zero mean and variance $\theta_2^2\sigma^2$, with $\theta_2 > \theta_1$, and

ϵ = the random error, statistically independent of t and distributed normally with zero mean and variance σ^2/n, with n = the sample size.

Moder, in (28), states "This model for ξ is quite appropriate in the paper, textile, chemical and electronic component industries where normal fluctuations in the raw material are both frequent and significant." Figure 4 represents this type of product quality model.

(3) When $w_1 = 0$, $w_2 = \pm k\sigma$, and $\mu(t) = \mu$ for $t \geq 0$ in the above example, Duncan (1), determines the $\overline{X}$-chart control limits, sample size and the inspection interval for maximum average net income for a process that fails as a result of chance causes.

Selecting a Quality Control Plan

For example consider $X(t;\xi)$ to be the normal distribution with mean μ and variance σ^2 for $t \leq t^*$, and mean $k\mu$, $k > 0$, and variance σ^2 for $t > t^*$. A natural choice for control of this process would be a $\overline{X}$-chart if k is large, say $k \geq 3$, or a cumulative sum chart (see (26) and (27)) when $k < 3$. The cumulative sum chart is more sensitive to small changes in the process mean (assuming $X(t;\xi)$ to be normal) than the $\overline{X}$-chart and economics will usually dictate the use of the former when $k < 3$. In the first example of the preceding section one might use a p-chart, or possibly an application of another type of control chart.

In order to choose a control plan for a particular $X(t;\xi)$ one must be familiar with the quality control literature. References (28) and (29) (both as yet unpublished) contain a summary of the literature pertaining to various control plans that have been applied to different representations of $X(t;\xi)$. These references also summarize the literature on economical design of quality control plans. Naturally, the criterion function, and hence the maximum expected total net income, will depend upon what quality control plan is used. The decision of what quality control plan to employ will not be investigated in this study.

Terminology

The following terms are introduced for this model.

Let t^* = the process failure arrival time, or Kh, whichever is the smaller,

"in-control" = the process activity when $t \leq t^*$,

"out-of-control" = the process activity when $t > t^*$,

H_o: = the statistical hypothesis that the process is in-control,

H_a: = the statistical hypothesis that the process is out-of-control,

α = the probability of concluding H_a: when H_o: is true, and is usually termed the type I error,

β = the probability of concluding H_o: when H_a: is true, usually called the type II error,

n = the sample size,

C_I = the cost in dollars for each type I error, or the cost of looking for a failure when none have occurred,

$\overline{I}_o$ = the expected number of times that the process is in-control when it is inspected, per tool cycle,

a = the fixed cost per sample associated with obtaining and processing the sample data,

b = the variable cost per unit of sampling, associated with obtaining and processing the data.

All the terms used in this Chapter are presented in the List of Symbols in Appendix L for easy reference.

The Criterion Function

It is assumed that $X(t;\xi)$ is a known function of its arguments and that statistical estimates of the distribution parameters are available. Let the quality control plan to be employed be called Q. The set Θ is defined to be a collection of independent (functionally independent and not statistically independent) variables each belonging to Q so that the elements of Θ completely determine Q. Therefore, the criterion function becomes

$$T_M = \underset{\{\Theta \cup \bar{h} \cup \bar{K}\}}{\text{Maximum}} \left[E\left\{\frac{\text{net income}}{\text{year}}\right\}\right], \tag{1}$$

where

$$\bar{h} = \left\{ h \middle| h \geq 0 \right\} \text{ and } \bar{K} = \left\{ K \middle| K = 1,2,\ldots \right\}.$$

Then, using equation (10) of Chapter III,

$$T_M = \underset{\{\Theta \cup \bar{h} \cup \bar{K}\}}{\text{Max}} \left[\sum_{\ell=1}^{4} P(\ell) \left\{ VE(U|\ell) - E(Uv|\ell) \right\} \right] - f_x, \tag{2}$$

where the "events ℓ" are defined in Chapter III. The quantities $P(\ell)$, $E(U|\ell)$, and $E(Uv|\ell)$ may be determined for this model by an analysis similar to that presented in Appendix K for Model II. The expressions for $E(U|\ell)$ and $E(Uv|\ell)$ will be far more complicated for this model than for Model II.

For example, if Q is a $\bar{X}$-chart with control limits L_1 and L_2, then Θ may be $\left\{\{n\}, \{L_1\}, \{L_2\}\right\}$, or $\left\{\{n\}, \{\alpha\}\right\}$, or $\left\{\{\alpha\}, \{\beta\}\right\}$. In other words, knowing α and β the sample size and control limits may be determined, and hence Q (in this case of an $\bar{X}$-chart) is determined. When Q is a cumulative sum chart Θ may be $\left\{\{n\}, \{\alpha\}, \{\beta\}\right\}$. Naturally, a judicious choice of Θ will facilitate the finding of the number T_M.

Assuming such a number T_M exists, then the values of the variables that produce this number will be called Θ_{op}, h_{op}, and K_{op}. In the following discussion it is assumed that Θ contains the three elements n, α, and β.

Development of the Criterion Function

The development of the criterion function for the General Model differs from that of Model II in the following ways:

(1) expected net income is a function of h, K, and some of the quality control plan variables (α, β, and n might be used),

(2) the inspection cost is related to the sample size and equals a + bn dollars per inspection,(*)

(3) the delay in stopping production when H_a: is accepted is a function of the sample size,

(4) the screening plan depends upon the nature of $X(t;\xi)$,

(5) the total number of units sold, U, during the period is affected by the screening plan,

(6) salvage income is a function of the number of units screened per failure and $X(t;\xi)$,

(7) a new cost is incurred, called the type I error cost, and equals $\alpha C_I I_o$ per tool cycle,(**)

(8) the number of inspections, total running hours, and out-of-control process running hours per tool cycle are functions of β, h, and K.

(*)Cowden (31) proposes that the inspection cost per tool cycle is of the form $C_1 + C_2 \cdot \bar{I} + C_3 \cdot n\bar{I}$, where C_1 is overhead, C_2 inspection cost, and C_3 a cost related to the sample size.

(**)Cowden (31) considers a fourth factor which increases when the shift in the process mean decreases, and decreases when the shift becomes larger. This factor is proportional to the cost of a search for the cause of failure. Cowden also considers a probability of finding the cause of defective production in his derivation of $\bar{H}_b$.

The criterion function is not derived in terms of its variables and parameters because the methods previously used to locate h_{op} and K_{op} will usually not be practical for this model.

Conclusions

The complexity of the criterion function will prohibit using differentiation as the method of locating the optimum operating conditions. Probably the most expedient method would be to employ simulation, together with "experimental design" procedures to minimize the number of simulation runs required to locate the optimum level of the model variables, Θ_{op}, h_{op}, and K_{op}.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Model I - Chance Failures

Model I is applicable to production systems where

(1) process failure is predominantly a result of chance causes, or

(2) wearout, without preventive maintenance, occurs and the process has been in operation for a "long time."

When there are n failure components, and component failures are mutually independent, the following results are applicable if:

(1) the value of the process failure arrival rate (λ) is the sum of the individual component's failure rate, and

(2) the expected value of each cost parameter is used.

The exact value of the optimum inspection interval (h_{op}) was calculated for various selections of the parameters and the results are in Tables 1 through 24. Approximate expressions for h_{op} are derived in Chapter II.

When the screening activity does not damage the product sequential screening should be used. If screening damages the product then the maximum net income value for continuous screening should be compared to the maximum net income value for sequential screening to decide on which plan is best (see Appendix E).

The unit sale price (V), salvage income (s), and total variable production cost per unit (Y) might be difficult to estimate. When the process under study is a part of a larger production line, V is the value of the product at this point in production. In this case it may be particularly difficult to set a value on V. If there is doubt as to what value Y,s, or V should be then use the smallest acceptable value for V or (Y-s) because the expected net income equation is relatively flat for $h > h_{op}$. In other words, a decrease in V or (Y-s) will cause h_{op} to increase and the resulting deviation from T_M will not be as great as if h_{op} decreased by the same amount.

Model II - Chance and Wearout Failures

The development of Model II depends upon the maintenance plan. If the maintenance plan outlined for Model II is accepted, then the maximum net income, say T_M^o, can be calculated. If another maintenance plan were used and the income equation were formulated using the criterion of maximizing net income, then the maximum net income, say T_M^*, may also be calculated using the same values of the parameters. The result may be $T_M^o < T_M^*$, or $T_M^o > T_M^*$. The question of the optimum maintenance plan was not investigated.

The maintenance plan used is applicable to systems that require a high degree of reliability and/or when it is economical to overhaul the process completely whenever it fails from wearout.

If other maintenance plans are studied the mathematical development of the income equation, and solution, may become extremely tedious. This is not a real drawback in studying other maintenance plans because, in any case, the resulting complexity of the criterion function will usually dictate that simulation be used to solve the problem.

Estimation of the risk parameters (λ, μ, N) is relatively easy except when only the times to a failure was discovered are known - a situation that usually occurs in practice. When this happens it is better to collect new failure data designed to give the time to a failure occurred than to try to estimate λ, μ, and N from the old data because an accurate failure density function is needed to obtain a reasonable degree of precision in h_{op} and K_{op}.

The study of this model for different maintenance plans and various other failure density functions is worthy of further research.

General Model

This model is probably the most important model for practical work because it incorporates both chance and wearout failures with the possibility for drifts and/or shifts in process quality. The selection of the quality control plan is of paramount importance because a change in the quality control plan (e.g. changing from a $\overline{X}$-chart to a cumulative sum chart) will change maximum net income. Probably the most expedient method of solving this problem for the optimum operating conditions would be to employ simulation.

To the author's knowledge this model has not been studied before and in his opinion it is deserving of further research.

General Conclusions

This study is primarily theoretical. The reader must pay particular attention to all the assumptions before applying the results. Here, attention is focused on the following three assumptions that are made in various parts of the study.

(1) "Wearout failure can be neglected;" the validity of this

assumption depends upon the wearout failure arrival rate and costs. If total production for the year is fixed, then when the total cost contributed through wearout is much smaller than that of chance failures, wearout might be neglected. On the other hand, if total production is not fixed, downtime costs are also relevant and should be studied before deciding that the effect of wearout can be omitted.

(2) In any application of the results it is recommended that the accuracy of the approximations used be checked -- especially those in equations (8), (14), and (17) of Chapter II. These equations appertain to "a long range study." The accuracy of one approximation, that of equation (8), is studied in Appendix C.

(3) Total production for the year will probably never be a predetermined constant. Assuming total production variable or fixed for the period of the study gives the two extremes of what will probably happen in practice.

A P P E N D I C E S

APPENDIX A

EXTENSION OF MODEL I TO THE CASE OF TWO OR MORE FAILURE COMPONENTS

Process Failure Density Function

Let the process contain n components whose failure results in defective production. The jth component of the machine may fail as a result of r_j different causes, for $j = 1,2,\ldots,n$. It is assumed that the various causes of machine tool failure occur independently for each of the n components and component failures are mutually independent. The arrivals of process failure for the jth component, as a result of the ith cause of chance failure, are considered to be at an average rate of λ_{ij} occurrences per failure free process running hour, for $i = 1,2,\ldots, r_j$ and $j = 1,2,\ldots n$.

Consider the ith cause of chance failure for the jth process failure component. The arrivals of process failure, for any $i = 1,2,\ldots, r_j$ and $j = 1,2,\ldots,n$, are considered to be random. This means that the chance of the next arrival of a failure is independent of the time since the last arrival of a failure. It can be shown (see (10) or (15)) that the probability density function describing this phenomena is

$$e_{ij}(t) = \begin{cases} \lambda_{ij}\, e^{-\lambda_{ij}t} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases} \tag{1}$$

where λ_{ij}^{-1} = the mean time between failures for the jth component, when a failure is a result of the ith cause of chance failure.

The probability density functions for the other $r_j - 1$ causes of chance failure, for the jth component, are similarly defined, for each $j = 1,2,\ldots, n$.

"Addition" of Two or More Failure Density Functions

First consider a process that has only one failure component. It is assumed that once this component fails, as a result of the occurrence of any one of the chance causes of failure, that the occurrence of another "failure" before an inspection does not affect the process in any way. For example, a cutting tool that first breaks will not be affected by the occurrence of failure that causes the tool to become chipped. Therefore, the distribution of the waiting time to the first failure will be the process failure distribution. Let $F(C)$ be the process failure distribution, for $C \geq 0$. Then

$$F(C) = P\left\{\text{a failure occurring by time } C\right\} =$$

$$1 - P\left\{\text{no failures to at least the time } C\right\}.$$

If T_i = the process running hours to a failure, when the failure was a result of the ith cause of chance failure, and $F_i(C)$ is the corresponding probability distribution function, for $i = 1,2,\ldots, M$, then

$$F(C) = 1 - \prod_{i=1}^{M} \left\{1 - F_i(C)\right\}, \tag{2}$$

if all the T_i are mutually independent.

It follows that if

$$f_i(t) = dF_i(t)/dt ,$$

for $i = 1,2,\ldots, M$, then

$$f(t) = \frac{dF(t)}{dt} = \sum_{i=1}^{M} \left\{ f_i(t) \prod_{\substack{j=1 \\ j \neq i}}^{M} \left[1 - F_j(t) \right] \right\} . \tag{3}$$

Equation (3) will give the failure density function for any process failure component. Consider the kth process failure component. Using equation (1) in equation (3), where $j = k$ and $i = 1,2,\ldots, r_k$, it is seen that the failure density function for the kth component is

$$e_k(t) = \begin{cases} \lambda_k e^{-\lambda_k t} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases} , \tag{4}$$

where $\lambda_k^{-1} = \left\{ \sum_{i=1}^{r_k} \lambda_{ik} \right\}^{-1}$ = the mean time between failures for the kth component.(*)

Now consider a process that contains n components whose failure results in defective production. Equations (4) and (3) may be used to give the distribution of the waiting time to the first process failure. The following assumption is made:

(1) only one component can fail within an inspection interval,

or

(2) the probability of two or more components failing within an

(*)Since the machine tool is fixed each time a failure is discovered, it can easily be shown that equation (4) is the failure density function for the time between the s and (s + 1)th failure, for any $s = 1,2,\ldots$.

inspection interval is very small and this effect is neglected.(*) Therefore, using equation (4), $k = 1,2,\ldots, n$, in equation (3) it is seen that the process failure density function is

$$e(t) = \begin{cases} \lambda e^{-\lambda t} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}, \qquad (5)$$

where $\lambda^{-1} = \left\{ \sum_{k=1}^{n} \lambda_k \right\}^{-1}$ = the mean time between process failures.(**)

This result means that the process will respond to its environment as a function of λ as if it does not know the values of the individual λ_{ij}, $i = 1,2,\ldots, r_j$ and $j = 1,2,\ldots, n$. The following section shows that the values of each λ_{ij} may be of importance.

The Criterion Function

Let s_{ij} = the salvage income for a defective produced when the jth component failed as a result of the ith cause of chance failure,

and similarly S_{ij}, R_{Cij}, and D_{ij} all pertain to when the jth component fails as a result of the ith cause of chance failure, for $i = 1,2,\ldots,r_j$

(*)This probability can be shown to be equal to

$$1 - \sum_{k=1}^{n} \frac{e^{\lambda_k h} - 1}{e^{\lambda h} - 1},$$

and is usually quite large if $n \geq 10$.

(**)Since the process is retooled for each failure, it can be shown, see (13), that equation (5) is the process failure density function for the time between the s and (s + 1)th process failure, for any $s=1,2,\ldots$.

and $j = 1,2,\ldots, n$.

Now consider

$$E\left\{\text{defective production salvage income}\right\} = E\left\{RsH_b\right\} = RE\left\{sH_b\right\} = R\sum_{i,j} P((ij))\ E\left(s_{ij}H_b \middle| (ij)\right) ,$$

where (ij) is the event that the process failed as a result of the occurrence of the ith cause of chance failure for the jth component, for $i = 1,2,\ldots, r_j$ and $j = 1,2,\ldots, n$.

It follows that (see Wilks (24) page 61)

$$E\left(s_{ij}H_b \middle| (ij)\right) = s_{ij}\int_0^\infty H_b\, dF\left(t \middle| (ij)\right) ,$$

where

$$F\left(t \middle| (ij)\right) = \int_0^t \frac{\lambda_{ij}\, e^{-\sum_{i,j}\lambda_{ij}X}\, dX}{\int_0^\infty \lambda_{ij} e^{-\sum_{i,j}\lambda_{ij}X}\, dX} = 1 - e^{-\sum_{i,j}\lambda_{ij}t} .$$

Hence

$$E\left\{sH_b \middle| (ij)\right\} = s_{ij}\,\overline{H}_b\,(h;\lambda) ,$$

and $\overline{H}_b$ is derived in Appendix D, where

$$\lambda = \sum_{i,j} \lambda_{ij} .$$

It is seen that

$$P((ij)) = \int_0^\infty \lambda_{ij} e^{-\lambda X}\, dX = \lambda_{ij}/\lambda \ ,$$

therefore

$$E\left\{RsH_b\right\} = R\overline{H}_b\ (h;\lambda) \sum_{i,j} \frac{s_{ij}\lambda_{ij}}{\lambda} \ .$$

Let

$$\bar{s} = E(s) = \sum_{i,j} \frac{s_{ij}\lambda_{ij}}{\lambda} \ , \qquad (6)$$

then

$$E(RsH_b) = RE(s)E(H_b) = R\bar{s}\ \overline{H}_b \ .$$

Using this same argument, it follows that

$$E\left\{\text{screening cost}\right\} = \overline{S}\ \overline{N}_o \ ,$$

$$E\left\{\text{Downtime cost}\right\} = C_d\overline{D} + \overline{R}_C \ ,$$

$$E\left\{\text{total clock hours per tool cycle}\right\} = \overline{L} + \overline{D} \ ,$$

where $\overline{S}$, $\overline{D}$, and $\overline{R}_C$ are defined as $\bar{s}$ is defined in equation (6).

When the process contains more than one failure component and/or a failure component fails as a result of more than one chance cause the criterion function for Model I changes in two ways. The changes are:

(1) Use $E(s)$, $E(S)$, $E(R_C)$, and $E(D)$ for s, S, R_C, and D respectively, and

(2) the expressions for $\overline{I}$, $\overline{H}_b$, $\overline{L}$, and $\overline{N}_o$ have λ as their parameter where $\lambda = \sum_{i,j} \lambda_{ij}$.

APPENDIX B

ESTIMATION OF THE MEAN TIME BETWEEN FAILURES, λ, FOR MODEL I

Define

t_i = the ith observation on the process running hours to a failure,

t_i^* = the ith observation on the process running time to when a failure is discovered,(*)

$\frac{1}{\lambda^*}$ = the mean process running time to when a failure is discovered,

$\bar{\lambda}^*$ = the statistical estimator of λ^*,

$\hat{\lambda}$ = the maximum likelihood estimator of λ,

$\bar{\lambda}$ = the estimator of λ when only the t_i^* values are available.

The maximum likelihood estimator is

$$\hat{\lambda} = (\bar{t})^{-1} = n\left\{\sum_{i=1}^{n} t_i\right\}^{-1} ,$$

which can be computed if the t_i numbers are available. A more practical estimator for λ, call it λ_o, is

$$\lambda_o = \frac{(n-1)}{n} \cdot \hat{\lambda} , \tag{1}$$

which is the only unbiased estimator for λ depending on $\bar{t}$ and it has less

---

(*) It is assumed that the lag time d, that it takes the inspector to decide that the process has failed, has already been subtracted to get the t_i^* numbers.

variance than the maximum likelihood estimator $\hat{\lambda}$.(*)

The estimator of λ^* will be taken as

$$\bar{\lambda}^* = (\bar{t}^*)^{-1} = n\left\{\sum_{i=1}^{n} t_i^*\right\}^{-1} . \qquad (2)$$

When only the t_i^* numbers are available, the most practical method of estimating λ would be by

$$\bar{\lambda} = \left\{(\bar{\lambda}^*)^{-1} - \bar{H}_b(\bar{\lambda};h)\right\}^{-1} . \qquad (3)$$

In Appendix D it is shown that(**)

$$\bar{H}_b\ (\lambda;h) = h(1-e^{-\lambda h})^{-1} - \lambda^{-1} . \qquad (4)$$

Substituting equations (2) and (4) into equation (3) gives

$$\bar{\lambda} = h^{-1}\ \ln\left\{\bar{t}^* / (\bar{t}^* - h)\right\} . \qquad (5)$$

It can be shown(***) that $\bar{\lambda}$ is a consistent estimator of λ.

(*) This result follows by noting that $\bar{t}$ is distributed according to the gamma probability law (see Wilks (24) p. 390).

(**) The lag time d has also been subtracted from $\bar{H}_b$.

(***) By observing that $(\bar{t}^*/h)$ is a consistent estimator for $\bar{I}$ and the logarithmic function is monotone and continuous.

APPENDIX C

ESTIMATING E(U) IN TERMS OF THE TOTAL NUMBER OF TOOL CYCLES m

Recall that

$$U = \frac{H_R R \sum_{i=1}^{m} T_i}{\sum_{i=1}^{m} (L_i + D)} , \tag{1}$$

and

$$E(U) \doteq \frac{H_R RE(T)}{E(L) + D} . \tag{2}$$

Let

$$\Omega_m = \frac{1}{m} \sum_{i=1}^{m} T_i , \tag{3}$$

$$\Phi_m = \frac{1}{m} \sum_{i=1}^{m} (L_i + D) = \frac{1}{m} \sum_{i=1}^{m} \left\{ h \left[\!\left[\frac{T_i}{h} + 1 \right]\!\right] + d + D \right\} = \frac{1}{m} \sum_{i=1}^{m} \phi_i . \tag{4}$$

It is easily seen that

$$E(T_i) = \lambda^{-1} ,$$

and from equation (5) of Appendix D

$$E(\phi_i) = E(L) + D = h\bar{I} + d + D = \phi \ . \tag{5}$$

It follows that

$$0 < Q^2 = \left\{ E\,\frac{\Omega_m}{\Phi_m} - \frac{E\Omega_m}{E\Phi_m} \right\}^2 =$$

$$= \left\{ E\left(\frac{\Omega_m E\Phi_m - \Phi_m E\Omega_m}{\Phi_m E\Phi_m} \right) \right\}^2$$

$$\leq E\left\{ \Omega_m \phi - \Phi_m \lambda^{-1} \right\}^2 \left[\frac{1}{\left\{E\Phi_m\right\}^2} \right] E(\Phi)_m^{-2} \ (*)$$

$$= E\left\{ (\Omega_m - \lambda^{-1}) - (\Phi_m - \phi)\,\frac{\lambda^{-1}}{\phi} \right\}^2 \cdot E(\Phi)_m^{-2}$$

$$= E\left[\sum_{i=1}^{m} \frac{(T_i - \lambda^{-1})}{m} - \frac{\lambda^{-1}}{\phi} \sum_{i=1}^{m} \frac{(\phi_i - \phi)}{m} \right]^2 \cdot E(\Phi_m)^{-2}$$

$$= mE\left\{ \left[\frac{T_1 - \lambda^{-1}}{m} \right] - \frac{\lambda^{-1}}{\phi}\, \frac{(\phi_1 - \phi)}{m} \right\}^2 \cdot E(\Phi)_m^{-2}$$

$$= \frac{1}{m}\, E\left\{ T_1 - \frac{\lambda^{-1}}{\phi} \cdot \phi_1 \right\}^2 \cdot E(\Phi)_m^{-2} \ . \tag{6}$$

From equation (4) it is noticed that

$$\phi_i \geq h + d + D \ ,$$

therefore

$$\Phi_m \geq h + d + D \ ,$$

(*) The Cauchy-Schwarz inequality for integrals.

hence

$$E(\phi_m)^{-2} \leq \left\{ h + d + D \right\}^{-2} .$$

An upper bound for the error in the approximation of equation (2) (equation (8) of Chapter II) is

$$H_R R \left| Q \right| = \left| E(U) - \frac{H_R R E(T)}{E(L) + D} \right| \leq$$

$$\frac{H_R R}{\sqrt{m}} \cdot \frac{\sqrt{E(T_1 \lambda \phi - \phi_1)^2}}{\lambda (E(L) + D)(h + d + D)} . \qquad (7)$$

The estimate of the upper bound for $|Q|$ may be improved by finding $E(\phi_m)^{-2}$, but the expression for $E(\phi_m)^{-2}$ is difficult to evaluate.

It is evident from equation (7) that, for fixed H_R, $H_R R|Q|$ can be made arbitrarily small for large m.

APPENDIX D

DERIVATION OF THE QUANTITIES $\overline{I}$, $\overline{L}$, AND $\overline{H}_b$ FOR MODEL I

Let

T = the process running hours to a failure, and

$$e(t) = \begin{cases} \lambda e^{-\lambda t} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases},$$

is the corresponding failure density function.

Recall that

H_b = the running hours of defective production per tool cycle,

L = total running hours per tool cycle, and

I = total inspections per tool cycle.

Then, it is seen that

$$I = \left[\!\left[\frac{T}{h} + 1 \right]\!\right], \tag{1}$$

$$L = h \cdot \left[\!\left[\frac{T}{h} + 1 \right]\!\right] + d = hI + d, \tag{2}$$

$$H_b = h \cdot \left[\!\left[\frac{T}{h} + 1 \right]\!\right] + d - T = hI + d - T. \tag{3}$$

Therefore

$$\overline{I} = E(I) = \int_0^{\infty} \left[\!\left[\frac{t}{h} + 1 \right]\!\right] \lambda e^{-\lambda t} dt =$$

$$= \lambda h \int_{o}^{\infty} \left[\!\left[s+1 \right]\!\right] e^{-\lambda hs}\, ds = \sum_{j=1}^{\infty} \lambda h \int_{(j-1)}^{j} je^{-\lambda hs}\, ds =$$

$$= \sum_{j=1}^{\infty} j \left\{ e^{-(\lambda h)(j-1)} - e^{-(\lambda h)j} \right\} = \left\{ 1 - e^{-\lambda h} \right\}^{-1} . \quad (4)$$

It immediately follows that

$$\overline{L} = h\overline{I} + d = h \left\{ 1 - e^{-\lambda h} \right\}^{-1} + d \ , \quad (5)$$

and

$$\overline{H}_b = h\overline{I} + d - E(T) = h \left\{ 1 - e^{-\lambda h} \right\}^{-1} + d - \lambda^{-1} . \quad (6)$$

APPENDIX E

DEVELOPMENT OF MODEL I WHEN THE SCREENING ACTIVITY DAMAGES THE PRODUCT

The two following assumptions are made:

(1) the screening and inspection activity cause the item to become defective, and

(2) there is only one failure component and a failure is the result of only one chance cause.

Terminology

Let

I_i = the number of inspections in the ith tool cycle,

L_i = the process running hours in the ith tool cycle,

$\rho_i(R\left\{h+d\right\})$ = the number of units screened in the ith tool cycle when sequential screening is used, $\rho(\ \ ) = E(\rho_i\left\{\ \right\})$, and

m = the number of tool cycles in the year.

Screening Procedure

It can be shown that in any practical case continuous screening forward in time (i.e. through the non-defectives) is uneconomical in this case. Therefore, continuous screening will be through the defectives and

$$\overline{N}_o = R\overline{H}_b \ . \tag{1}$$

Development of the Criterion Function

It is noted that now

$$U = \left\{ \frac{\text{total number of non-defectives produced}}{\text{year}} \right\} - \left\{ \frac{\text{total number of non-defectives destroyed}}{\text{year}} \right\} . \quad (2)$$

Let

$$\theta = \frac{\text{the number of non-defectives destroyed}}{\text{plant operating clock hour}} , \quad (3)$$

and if screening is continuous, then

$$\theta = \frac{\sum_{i=1}^{m} I_i}{\sum_{i=1}^{m} (L_i + D)} , \quad (4)$$

because one good item is destroyed at each inspection.

The following approximation for $E(\theta)$ will be used(*)

$$E(\theta) \doteq \frac{\overline{I}}{\overline{L} + D} , \quad (5)$$

therefore

$$E(U) \doteq \frac{H_R}{\overline{L} + D} \left\{ \frac{R}{\overline{\lambda}} - \overline{I} \right\} . \quad (6)$$

If sequential mid-range screening is used, then

(*)See footnote page 13.

$$\theta = \frac{\sum_{i=1}^{m} \left\{ (I_i - 1) + W_i \right\}}{\sum_{i=1}^{m} (L_i + D)} , \tag{7}$$

where

W_i = the number of non-defective units destroyed as a result of sequential screening in the ith tool cycle.

It is reasonable to estimate W_i by $\rho(R(h+d))/2$.(*) The exact formula for W_i is complex and cumbersome and it is not considered to be warranted in this study.

Therefore, $W_i \doteq \rho_i(R \left\{h+d\right\})/2$ and

$$\overline{W} = E(W_i) \doteq \rho(R \left\{h+d\right\})/2 .$$

Hence, the following approximation will be used(**)

$$E(\theta) \doteq \frac{\overline{I} - 1 + \frac{\rho(R \left\{h+d\right\})}{2}}{\overline{L} + D} , \tag{8}$$

and

$$E(U) \doteq \frac{H_R}{\overline{L} + D} \left\{ \frac{R}{\lambda} - \overline{I} + 1 - \frac{\rho(R \left\{h+d\right\})}{2} \right\} . \tag{9}$$

(*) It can be demonstrated (see (16)) that if $\lambda h \leq 1$, a condition that holds in any practical case, the exponential failure density function is very close to the uniform density function between any two points kh and $(k + 1)\cdot h$.

(**) See footnote page 13.

Derivation of E(v)

When screening is continuous and through the defectives it can be shown (consult the section on the derivation of E(v)) that

$$E(v) \doteq \frac{\lambda}{R}\left\{C\bar{I} + R(Y-s)\bar{H}_b + SR\bar{H}_b - s\bar{I} + C_d D + R_C\right\} + Y , \qquad (10)$$

because only one non-defective is damaged at each inspection.

If sequential mid-range seach is used then

$$E(v) \doteq \frac{\lambda}{R}\left\{C\bar{I} + R(Y-s)\bar{H}_b + S\rho(R\{h+d\}) - \frac{s\rho(R\{h+d\})}{2} - s(\bar{I}-1) + C_d D + R_C\right\} + Y . \qquad (11)$$

Comparison of Continuous and Sequential Screening

The decision of which screening plan to use depends upon their respective T_M values. Obviously, the plan to use is the one that has the largest T_M.

Let

$(T_M)_1$ = the maximum expected net income when continuous screening is used,

$h_1 = h_{op}$, where h_{op} produces $(T_M)_1$,

$(T_M)_2$ = the maximum expected net income when sequential mid-range search is used, and

$h_2 = h_{op}$, where h_{op} produces $(T_M)_2$.

Therefore, use sequential mid-range search if

$$(T_M)_2 > (T_M)_1 ,$$

or(*)

(*)This follows from first deriving E(Uv) and then forming the equations for T_M (see equation (19) of Chapter II) and then making the comparison.

$$s + V\left\{\frac{\rho(R\left\{h_2+d\right\})}{2} - 1\right\} + \rho(R\left\{h_2+d\right\})\left\{S - \frac{s}{2}\right\}$$

$$< RS \cdot \overline{H}_b\ (h_1\ ;\ \lambda)\ ,$$

otherwise use continuous screening through the defectives.

The TRI Equations - U Variable

When screening is continuous h_{op} is the positive zero of

$$\frac{d}{dh}\left\{\frac{(C + V - s)\overline{I} + R(Y - s + S)\overline{H}_b - C_S}{h\overline{I} + d + D}\right\} = 0\ . \qquad (12)$$

It is seen that this equation is of the same form as the TRI_1 equation. The general solution of equation (12) is contained in the general solution of the TRI_1 equation.(*)

When sequential screening is used h_{op} will be determined by the h coordinate at the minimum of the following expression

$$\frac{(V + C - s)\overline{I} + R(Y-s)\ \overline{H}_b + \rho(R\cdot(h+d))\cdot\left\{S + \frac{V-s}{2}\right\} - (V - s + C_S)}{h\overline{I} + d + D}\ . \qquad (13)$$

Equation (13) is of the same form as the TRI_2 equation and h_{op} would be determined by the procedure outlined for TRI_2.

The TRC Equations - U Constant

When screening is continuous h_{op} is the positive zero of

$$\frac{d}{dh}\left\{(C - s)\overline{I} + R(Y - s + S)\overline{H}_b\right\} = 0\ . \qquad (14)$$

It is seen that the general solution of equation (14) is contained in the general solution for the TRC_1 equation.

(*)This is seen by differentiating equation (12) and comparing to equation (39) of Chapter II.

When sequential mid-range search is used then h_{op} will be the h coordinate at the minimum of the following expression

$$(C - s)\bar{I} + R(Y - s)\bar{H}_b + \rho(R\left\{h+d\right\})(S - \frac{s}{2}) . \qquad (15)$$

Equation (15) is of the same form as the TRC_2 equation and h_{op} would be determined by the same procedure outlined for TRC_2.

APPENDIX F

INSPECTION INTERVALS FOR MAXIMUM INCOME - CONTINUOUS SCREENING AND CHANCE FAILURES

The Tables in this Appendix give the optimum inspection interval (h_{op}) and TRI_1^* ($h = h_{op}$) for selections of λ, $\alpha_o = d + D$, and the cost ratios A, B, and C_{S1}. The Tables also contain the per cent increase in TRI_1^* (and hence the per cent decrease in T_M) for four values of h. For example, if $B = 1.0$, $C_{S1} = 1000.0$, $A = 0.9$, $\lambda = 0.01$, and $\alpha_o = 0.1$ (see Table 1) then $h_{op} = 4.497$ hours, $TRI_1^* = -8.64$, and

$$TRI_1^* \ (h = (0.115)(4.497)) = -8.64 \ (1.0 - 0.1747),$$

$$TRI_1^* \ (h = (0.535)(4.497)) = -8.64 \ (1.0 - 0.0103),$$

$$TRI_1^* \ (h = (2.0)(4.497)) = -8.64 \ (1.0 - 0.0124),$$

$$TRI_1^* \ (h = (5.0)(4.497)) = -8.64 \ (1.0 - 0.0763).$$

On the other hand, when $TRI_1^* \doteq 0$ maximum net income (T_M) decreases rapidly if $h < h_{op}$. For example, if $B = 0.01$, $C_{S1} = 10.0$, $A = 1.0$, $\lambda = 0.1$, and $\alpha_o = 1.0$ (see Table 8) then $h_{op} = 0.452$ hours, $TRI_1^* = 0.04$ and

$$TRI_1^* \ (h = (0.1)(0.452)) = 0.04(1.0 + 4.0128),$$

$$TRI_1^* \ (h = (0.5)(0.452)) = 0.04(1.0 + 0.2463),$$

$$TRI_1^* \ (h = (1.5)(0.452)) = 0.04(1.0 + 0.0811),$$

$$TRI_1^* \ (h = (1.9)(0.452)) = 0.04(1.0 + 0.2064).$$

Table 1. Inspection Intervals for Maximum Income, Continuous Screening and B = 1.0, C_{S1} = 1000.0

A	λ(failures/hour)	α_o (hours)	(0.115) $\cdot h_{op}$ (%) (*)	(0.535) $\cdot h_{op}$ (%) (*)	Inspection Interval (hours)	(2.0) $\cdot h_{op}$ (%) (*)	(5.0) $\cdot h_{op}$ (%) (*)	$-TRI_1^*$
0.5	1.0	10.0	4.79	0.28	0.144	0.36	2.40	89.58
		1.0	11.06	0.65	0.064	0.80	5.05	483.84
		0.1	14.94	0.88	0.048	1.06	6.57	870.07
	0.1	10.0	13.38†	0.82††	0.628	0.69‡	-	48.14
		1.0	17.96†	1.10††	0.473	0.92‡	-	86.58
		0.1	18.74†	1.15††	0.454	0.95‡	-	94.12
	0.01	10.0	16.68	0.98	4.503	1.19	7.33	8.23
		1.0	17.30	1.02	4.347	1.23	7.56	8.95
		0.1	17.37	1.02	4.330	1.24	7.59	9.02
0.9	1.0	10.0	4.77	0.28	0.145	0.36	2.40	89.54
		1.0	11.07	0.65	0.064	0.80	5.05	483.65
		0.1	14.95	0.88	0.048	1.06	6.56	869.72
	0.1	10.0	13.35†	0.82††	0.632	0.69‡	-	47.95
		1.0	17.97†	1.10††	0.474	0.92‡	-	86.23
		0.1	18.75†	1.15††	0.456	0.95‡	-	93.74
	0.01	10.0	16.74	0.98	4.683	1.19	7.36	7.88
		1.0	17.40	1.02	4.515	1.24	7.60	8.57
		0.1	17.47	1.03	4.497	1.24	7.63	8.64
1.0	1.0	10.0	5.68†	0.35††	0.145	0.30‡	-	89.53
		1.0	13.11†	0.80††	0.064	0.69‡	-	483.60
		0.1	17.89†	1.10††	0.047	0.89‡	-	869.64
	0.1	10.0	13.35†	0.82††	0.633	0.69‡	-	47.90
		1.0	17.96†	1.10††	0.475	0.92‡	-	86.14
		0.1	18.74†	1.14††	0.456	0.96‡	-	93.64
	0.01	10.0	19.94†	1.22††	4.731	1.02‡	-	7.79
		1.0	20.72†	1.27††	4.560	1.06‡	-	8.47
		0.1	20.80†	1.27††	4.542	1.06‡	-	8.55

(*) The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

† $h = (0.1)h_{op}$

†† $h = (0.5)h_{op}$

‡ $h = (1.9)h_{op}$

Table 2. Inspection Intervals for Maximum Income, Continuous Screening and B = 1.0, $C_{S1} = 500.0$

A	λ(failures/hour)	α_o (hours)	(0.115) · h_{op} (%) (*)	(0.535) · h_{op} (%) (*)	Inspection Interval (hours)	(2.0) · h_{op} (%) (*)	(5.0) · h_{op} (%) (*)	$-TRI_1^*$
0.5	1.0	10.0	6.92	0.41	0.202	0.52	3.53	44.48
		1.0	15.86	0.93	0.090	1.14	7.13	238.46
		0.1	21.43†	1.26††	0.068	1.51†††	9.14‡	426.87
	0.1	10.0	19.49†	1.19††	0.880	1.00†††	2.71‡	23.60
		1.0	26.01†	1.59††	0.669	1.32†††	3.52‡	42.25
		0.1	27.11†	1.65††	0.643	1.37†††	3.66‡	45.90
	0.01	10.0	26.73	1.57	6.098	1.89	11.43	3.79
		1.0	27.60	1.62	5.918	1.94	11.71	4.12
		0.1	27.69	1.62	5.900	1.95	11.74	4.15
0.9	1.0	10.0	6.89	0.41	0.202	0.52	3.53	44.44
		1.0	15.84	0.93	0.090	1.14	7.14	238.27
		0.1	21.39†	1.25††	0.068	1.51†††	9.16‡	426.53
	0.1	10.0	19.41†	1.19††	0.890	1.00†††	2.71‡	23.41
		1.0	26.01†	1.59††	0.674	1.32†††	3.52‡	41.91
		0.1	27.13†	1.66††	0.648	1.37†††	3.65‡	45.52
	0.01	10.0	27.30	1.60	6.562	1.93	11.67	3.45
		1.0	28.24	1.66	6.360	1.99	11.97	3.74
		0.1	28.34†	1.66††	6.339	1.99†††	12.01‡	3.78
1.0	1.0	10.0	8.18†	0.50††	0.203	0.44†††	1.23‡	44.44
		1.0	18.87†	1.16††	0.090	0.97†††	2.63‡	238.23
		0.1	25.56†	1.57††	0.067	1.28†††	3.43‡	426.44
	0.1	10.0	19.40†	1.19††	0.893	1.00†††	2.71‡	23.36
		1.0	26.00†	1.59††	0.676	1.32†††	3.52‡	41.82
		0.1	27.13†	1.66††	0.649	1.37†††	3.65‡	45.43
	0.01	10.0	32.64†	1.99††	6.696	1.65†††	4.42‡	3.36
		1.0	33.78†	2.06††	6.486	1.71†††	4.55‡	3.65
		0.1	33.90†	2.07††	6.464	1.71†††	4.57‡	3.68

(*) The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

† $h = (0.1)h_{op}$

†† $h = (0.5)h_{op}$

††† $h = (1.9)h_{op}$

‡ $h = (2.8)h_{op}$

Table 3. Inspection Intervals for Maximum Income, Continuous Screening and B = 1.0, C_{S1} = 100.0

A	λ(failures/hour)	α_o (hours)	(0.115)·h_{op} (%) (*)	(0.535)·h_{op} (%) (*)	Inspection Interval (hours)	(2.0)·h_{op} (%) (*)	(5.0)·h_{op} (%) (*)	$-TRI_1^*$
0.5	1.0	10.0	17.37	1.04	0.42	1.34	9.00	8.58
		1.0	37.83	2.22	0.20	2.66	15.82	44.61
		0.1	50.98	2.96	0.15	3.43	19.29	78.35
	0.1	10.0	52.84†	3.23††	1.83	3.12	18.49	4.18
		1.0	67.74†	4.10††	1.47	3.83	21.56	7.37
		0.1	70.34†	4.25††	1.42	3.95	22.05	7.80
	0.01	10.0	210.82	12.30	10.52	14.36	81.94	0.28
		1.0	213.01	12.40	10.45	14.41	81.49	0.30
		0.1	213.24	12.42	10.44	14.41	81.44	0.30
0.9	1.0	10.0	17.17	1.03	0.42	1.34	9.02	8.55
		1.0	37.74	2.22	0.20	2.66	15.82	44.44
		0.1	50.92	2.96	0.15	3.44	19.31	78.04
	0.1	10.0	52.55†	3.21††	1.92	3.11	18.58	4.01
		1.0	68.06†	4.12††	1.53	3.85	21.67	7.05
		0.1	70.77†	4.28††	1.48	3.98	22.17	7.63
	0.01	10.0	971.54	56.60	13.52	65.75	371.65	-0.05
		1.0	974.26	56.63	13.55	65.30	364.47	-0.05
		0.1	974.55	56.63	13.55	65.25	363.70	-0.05
1.0	1.0	10.0	20.33†	1.27††	0.42	1.34	9.03	8.54
		1.0	44.85†	2.74††	0.20	2.66	15.83	44.40
		0.1	60.50†	3.65††	0.16	3.43	19.31	77.96
	0.1	10.0	52.46†	3.21††	1.94	3.11	18.60	3.97
		1.0	68.11†	4.12††	1.54	3.86	21.70	6.97
		0.1	70.87†	4.28††	1.49	3.98	22.20	7.55
	0.01	10.0	393.31†	23.85††	14.76	22.37	126.48	-0.13
		1.0	393.02†	23.78††	14.84	22.11	-	-0.14
		0.1	392.97†	23.77††	14.85	22.08	-	-0.14

(*)The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

† $h = (0.1)h_{op}$

†† $h = (0.5)h_{op}$

Table 4. Inspection Intervals for Maximum Income, Continuous Screening and B = 1.0, C_{S1} = 10.0

A	λ(failures/hour)	α_o (hours)	(0.2)·h_{op} (%) (*)	(0.6)·h_{op} (%) (*)	Inspection Interval (hours)	(2.0)·h_{op} (%) (*)	(4.0)·h_{op} (%) (*)	$-TRI_1^*$
0.5	1.0	10.0	53.86	4.50	0.88	8.40	35.70	0.62
		1.0	87.31	7.01	0.61	11.38	41.39	2.93
		0.1	112.39	8.73	0.52	13.08	44.30	4.80
	0.1	10.0	897.83	72.63	3.62	122.46	472.39	-0.05
		1.0	936.41	73.57	3.67	113.37	395.65	-0.08
		0.1	944.14	73.78	3.68	111.80	383.87	-0.09
	0.01	10.0	40.95	3.32	14.11	5.71	22.78	-0.50
		1.0	40.21	3.25	14.42	5.53	21.76	-0.54
		0.1	40.13	3.24	14.46	5.51	21.65	-0.54
0.9	1.0	10.0	52.96	4.48	0.93	8.60	37.22	0.60
		1.0	87.33	7.02	0.64	11.50	41.95	2.80
		0.1	113.23	8.80	0.54	13.19	44.66	4.58
	0.1	10.0	182.84	14.85	4.42	25.32	98.48	-0.20
		1.0	180.98	14.13	4.76	21.39	73.35	-0.33
		0.1	180.59	14.00	4.84	20.70	69.51	-0.35
	0.01	10.0	13.92	1.11	25.51	1.82	6.81	-0.80
		1.0	12.33	0.97	28.89	1.52	5.44	-0.86
		0.1	12.14	0.95	29.34	1.49	5.28	-0.87
1.0	1.0	10.0	52.70	4.47	0.94	8.66	37.65	0.59
		1.0	87.44	7.04	0.65	11.53	42.11	2.77
		0.1	113.43	8.81	0.55	13.22	44.76	4.52
	0.1	10.0	145.96	11.89	4.70	20.43	79.94	-0.23
		1.0	141.47	11.05	5.20	16.72	57.30	-0.38
		0.1	140.51	10.87	5.30	16.06	53.85	-0.41
	0.01	10.0	8.84	0.70	36.91	1.13	4.18	-0.87
		1.0	6.49	0.50	50.41	0.65†	-	-0.93
		0.1	6.18	0.47	52.88	0.61†	-	-0.94

(*)The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

† $h = (1.9)h_{op}$

Table 5. Inspection Intervals for Maximum Income, Continuous Screening and B = 0.01, C_{S1} = 1000.0

A	λ(failures/hour)	α_o (hours)	(0.115)·h_{op} (%) (*)	(0.535)·h_{op} (%) (*)	Inspection Interval (hours)	(2.0)·h_{op} (%) (*)	(5.0)·h_{op} (%) (*)	$-TRI_1^*$
0.5	1.0	10.0	0.46	0.02	0.0147	0.03	0.22	90.74
		1.0	1.07	0.06	0.0063	0.07	0.50	498.17
		0.1	1.47	0.08	0.0046	0.10	0.67	904.76
	0.1	10.0	1.30†	0.08††	0.0616	0.06‡	-	49.59
		1.0	1.74†	0.10††	0.0457	0.09‡	-	90.07
		0.1	1.81†	0.11††	0.0439	0.09‡	-	98.07
	0.01	10.0	1.61	0.09	0.4449	0.11	0.75	8.60
		1.0	1.67	0.09	0.4288	0.12	0.78	9.36
		0.1	1.68	0.10	0.4272	0.12	0.78	9.44
0.9	1.0	10.0	0.46	0.02	0.0147	0.03	0.21	90.70
		1.0	1.07	0.06	0.0063	0.07	0.50	497.97
		0.1	1.47	0.08	0.0046	0.10	0.67	904.40
	0.1	10.0	1.28†	0.07††	0.0622	0.07‡	-	49.40
		1.0	1.72†	0.10††	0.0463	0.09‡	-	89.70
		0.1	1.82†	0.11††	0.0439	0.09‡	-	97.68
	0.01	10.0	1.62	0.09	0.4626	0.11	0.75	8.23
		1.0	1.68	0.10	0.4455	0.12	0.78	8.96
		0.1	1.69	0.10	0.4436	0.12	0.79	9.04
1.0	1.0	10.0	0.53†	0.03††	0.0150	0.03‡	-	90.70
		1.0	1.25†	0.07††	0.0064	0.07‡	-	497.92
		0.1	1.78†	0.11††	0.0046	0.08‡	-	904.32
	0.1	10.0	1.31†	0.08††	0.0616	0.06‡	-	49.34
		1.0	1.73†	0.10††	0.0463	0.09‡	-	89.61
		0.1	1.81†	0.11††	0.0439	0.09‡	-	97.60
	0.01	10.0	1.92†	0.12††	0.4625	0.10‡	-	8.14
		1.0	2.00†	0.12††	0.4473	0.10‡	-	8.87
		0.1	2.02†	0.12††	0.4457	0.10‡	-	8.95

(*) The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

† $h = (0.1)h_{op}$

†† $h = (0.5)h_{op}$

‡ $h = (1.9)h_{op}$

Table 6. Inspection Intervals for Maximum Income, Continuous Screening and B = 0.01, $C_{S1} = 500.0$

A	λ(failures/hour)	α_o	$(0.115)\cdot h_{op}$	$(0.535)\cdot h_{op}$	Inspection Interval	$(2.0)\cdot h_{op}$	$(5.0)\cdot h_{op}$	$-TRI_1^*$
		(hours)	(%) (*)	(%) (*)	(hours)	(%) (*)	(%) (*)	
0.1	1.0	10.0	0.66	0.03	0.0205	0.04	0.31	45.32
		1.0	1.53	0.09	0.0089	0.11	0.71	248.63
		0.1	2.06	0.12	0.0066	0.15	0.96	451.35
	0.1	10.0	1.87†	0.11††	0.0871	0.10†††	0.27‡	24.63
		1.0	2.51†	0.15††	0.0651	0.13†††	0.35‡	44.72
		0.1	2.60†	0.16††	0.0632	0.13†††	0.37‡	48.69
	0.01	10.0	2.54	0.15	0.5993	0.18	1.18	4.06
		1.0	2.62	0.15	0.5807	0.19	1.22	4.42
		0.1	2.63	0.15	0.5788	0.19	1.22	4.46
0.9	1.0	10.0	0.66	0.03	0.0207	0.04	0.31	45.28
		1.0	1.53	0.09	0.0089	0.11	0.71	248.43
		0.1	2.06	0.12	0.0066	0.15	0.96	451.00
	0.1	10.0	1.87†	0.11††	0.0877	0.09†††	0.26‡	24.43
		1.0	2.50†	0.15††	0.0657	0.13†††	0.35‡	44.36
		0.1	2.60†	0.16††	0.0633	0.13†††	0.37‡	48.30
	0.01	10.0	2.59	0.15	0.6449	0.18	1.20	3.70
		1.0	2.68	0.15	0.6238	0.19	1.24	4.03
		0.1	2.69	0.15	0.6217	0.19	1.25	4.06
1.0	1.0	10.0	0.79†	0.05††	0.0204	0.03†††	0.10‡	45.28
		1.0	1.84†	0.11††	0.0089	0.09†††	0.25‡	248.38
		0.1	2.57†	0.16††	0.0064	0.11†††	0.32‡	450.89
	0.1	10.0	1.87†	0.11††	0.0883	0.09†††	0.26‡	24.39
		1.0	2.51†	0.15††	0.0657	0.12†††	0.35‡	44.28
		0.1	2.60†	0.16††	0.0633	0.13†††	0.37‡	48.20
	0.01	10.0	3.10†	0.19††	0.6572	0.16†††	0.44‡	3.61
		1.0	3.19†	0.19††	0.6359	0.16†††	0.45‡	3.93
		0.1	3.22†	0.20††	0.6328	0.16†††	0.45‡	3.96

(*) The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

† $h = (0.1)h_{op}$

†† $h = (0.5)h_{op}$

††† $h = (1.9)h_{op}$

‡ $h = (2.8)h_{op}$

Table 7. Inspection Intervals for Maximum Income, Continuous Screening and B = 0.01, C_{S1} = 100.0

A	λ(failures/hour)	α_o (hours)	(0.115)·h_{op} (%) (*)	(0.535)·h_{op} (%) (*)	Inspection Interval (hours)	(2.0)·h_{op} (%) (*)	(5.0)·h_{op} (%) (*)	$-TRI_1^*$
0.5	1.0	10.0	1.58	0.09	0.043	0.11	0.76	9.00
		1.0	3.50	0.20	0.020	0.25	1.62	49.24
		0.1	4.65†	0.27††	0.015	0.34†††	2.16‡	89.22
	0.1	10.0	4.81†	0.29††	0.179	0.25†††	0.68‡	4.69
		1.0	6.12†	0.37††	0.141	0.32†††	0.86‡	8.51
		0.1	6.34†	0.38††	0.136	0.33†††	0.90‡	9.26
	0.01	10.0	13.96	0.82	1.015	1.01	6.43	0.44
		1.0	14.10	0.83	1.005	1.02	6.49	0.48
		0.1	14.12	0.83	1.004	1.02	6.49	0.48
0.9	1.0	10.0	1.56	0.09	0.044	0.11	0.74	8.97
		1.0	3.48	0.20	0.020	0.25	1.62	49.04
		0.1	4.68†	0.27††	0.015	0.34†††	2.15‡	88.86
	0.1	10.0	4.77†	0.29††	0.187	0.25†††	0.67‡	4.50
		1.0	6.15†	0.37††	0.147	0.32†††	0.87‡	8.15
		0.1	6.39†	0.39††	0.141	0.33†††	0.90‡	8.87
	0.01	10.0	61.84	3.66	1.301	4.50	28.47	0.08
		1.0	62.04	3.67	1.298	4.51	28.50	0.08
		0.1	62.07†	3.67††	1.297	4.51†††	28.50‡	0.08
1.0	1.0	10.0	1.86†	0.11††	0.044	0.09†††	0.26‡	8.96
		1.0	4.16†	0.25††	0.020	0.21†††	0.58‡	49.00
		0.1	5.53†	0.33††	0.015	0.29†††	0.79‡	88.78
	0.1	10.0	4.77†	0.29††	0.190	0.24†††	0.67‡	4.45
		1.0	6.17†	0.38††	0.148	0.32†††	0.86‡	8.06
		0.1	6.41†	0.39††	0.142	0.33†††	0.90‡	8.77
	0.01	10.0	403.92†	24.95††	1.419	21.09†††	57.08‡	-1.28
		1.0	403.51†	24.88††	1.420	21.07†††	56.98‡	-1.40
		0.1	403.53†	24.88††	1.420	21.06†††	56.96‡	-1.41

(*) The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

† $h = (0.1)h_{op}$

†† $h = (0.5)h_{op}$

††† $h = (1.9)h_{op}$

‡ $h = (2.8)h_{op}$

Table 8. Inspection Intervals for Maximum Income, Continuous Screening and B = 0.01, $C_{S1} = 10.0$

A	λ(failures/hour)	α_o (hours)	(0.1)·h_{op} (%) (*)	(0.5)·h_{op} (%) (*)	Inspection Interval (hours)	(1.5)·h_{op} (%) (*)	(1.9)·h_{op} (%) (*)	$-TRI_1^*$
0.5	1.0	10.0	9.50	0.58	0.092	0.20	0.51	0.84
		1.0	15.50	0.95	0.057	0.32	0.81	4.57
		0.1	19.93	1.23	0.045	0.39	1.00	8.23
	0.1	10.0	53.96	3.32	0.340	1.10	2.81	0.22
		1.0	57.06	3.50	0.323	1.15	2.93	0.40
		0.1	57.68	3.54	0.320	1.16	2.96	0.43
	0.01	10.0	14.61	0.90	1.334	0.29	0.76	-0.38
		1.0	14.40	0.88	1.352	0.29	0.74	-0.41
		0.1	14.37	0.88	1.355	0.29	0.74	-0.41
0.9	1.0	10.0	9.01	0.55	0.101	0.19	0.48	0.81
		1.0	15.34	0.93	0.060	0.32	0.81	4.38
		0.1	20.00	1.23	0.047	0.39	1.01	7.88
	0.1	10.0	372.24	22.92	0.417	7.60	19.41	0.02
		1.0	378.36	23.21	0.413	7.65	19.47	0.05
		0.1	379.48	23.26	0.413	7.67	19.50	0.05
	0.01	10.0	4.28	0.26	2.335	0.08	0.22	-0.74
		1.0	3.88	0.23	2.576	0.07	0.20	-0.80
		0.1	3.83	0.23	2.605	0.07	0.19	-0.81
1.0	1.0	10.0	8.90	0.55	0.103	0.18	0.47	0.80
		1.0	15.37	0.94	0.061	0.31	0.80	4.33
		0.1	19.94	1.22	0.047	0.40	1.02	7.79
	0.1	10.0	401.83	24.77	0.448	8.22	20.99	-0.02
		1.0	401.28	24.63	0.452	8.11	20.64	-0.04
		0.1	401.46	24.65	0.453	8.07	20.54	-0.04
	0.01	10.0	2.66	0.16	3.345	0.05	0.13	-0.82
		1.0	2.05	0.12	4.343	0.04	0.10	-0.90
		0.1	1.97	0.12	4.517	0.03	0.10	-0.90

(*) The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

Table 9. Inspection Intervals for Maximum Income, Continuous Screening and B = 0.0001, C_{S1} = 1000.0

A	λ(failures/hour)	α_o	(0.115) $\cdot h_{op}$	(0.535) $\cdot h_{op}$	Inspection Interval	(2.0) $\cdot h_{op}$	(5.0) $\cdot h_{op}$	$-TRI_1^*$
		(hours)	(%) (*)	(%) (*)	(hours)	(%) (*)	(%) (*)	
0.5	1.0	10.0	0.04	0.00†	0.0014	0.00†	0.02	90.85
		1.0	0.09	0.00†	0.0007	0.00†	0.05	499.59
		0.1	0.12	0.00†	0.0005	0.01	0.08	908.24
	0.1	10.0	-	0.00†	0.0054	0.00†	-	49.73
		1.0	-	0.00†	0.0041	0.01	-	90.43
		0.1	-	0.00†	0.0039	0.01	-	98.49
	0.01	10.0	0.16	0.00†	0.0445	0.01	0.07	8.63
		1.0	0.16	0.00†	0.0428	0.01	0.07	9.40
		0.1	0.16	0.00†	0.0426	0.01	0.07	9.48
0.9	1.0	10.0	0.04	0.00†	0.0014	0.00†	0.02	90.81
		1.0	0.09	0.00†	0.0007	0.00†	0.05	499.39
		0.1	0.12	0.00†	0.0005	0.01	0.08	907.88
	0.1	10.0	-	0.00†	0.0055	0.01	-	49.54
		1.0	-	0.00†	0.0042	0.01	-	90.05
		0.1	-	0.01	0.0039	0.01	-	98.07
	0.01	10.0	0.16	0.00†	0.0462	0.01	0.07	8.27
		1.0	0.16	0.00†	0.0445	0.01	0.07	9.00
		0.1	0.16	0.01	0.0443	0.01	0.07	9.09
1.0	1.0	10.0	-	-	0.0014	-	-	90.81
		1.0	-	-	0.0006	-	-	499.30
		0.1	-	-	0.0005	-	-	907.74
	0.1	10.0	-	-	0.0055	-	-	49.49
		1.0	-	-	0.0042	-	-	89.98
		0.1	-	-	0.0039	-	-	97.99
	0.01	10.0	-	-	0.0467	-	-	8.18
		1.0	-	-	0.0453	-	-	8.91
		0.1	-	-	0.0452	-	-	8.99

(*) The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

† No change in the first four digets.

Table 10. Inspection Intervals for Maximum Income, Continuous Screening and B = 0.0001, C_{S1} = 500.0

A	λ(failures/hour)	α_o (hours)	(0.115) · h_{op} (%) (*)	(0.535) · h_{op} (%) (*)	Inspection Interval (hours)	(2.0) · h_{op} (%) (*)	(5.0) · h_{op} (%) (*)	$-TRI_1^*$
0.5	1.0	10.0	0.06	0.00‡	0.0020	0.00‡	0.03	45.40
		1.0	0.16	0.01	0.0008	0.00‡	0.06	249.64
		0.1	0.19	0.01	0.0007	0.01	0.10	453.82
	0.1	10.0	0.15†	0.00†† ‡	0.0075	0.02†††	-	24.74
		1.0	0.29†	0.00†† ‡	0.0046	0.04†††	-	44.97
		0.1	0.29†	0.00†† ‡	0.0041	0.05†††	-	48.98
	0.01	10.0	0.25	0.01	0.0599	0.01	0.11	4.09
		1.0	0.26	0.01	0.0579	0.01	0.12	4.45
		0.1	0.26	0.01	0.0578	0.01	0.12	4.49
0.9	1.0	10.0	0.06	0.00‡	0.0020	0.00‡	0.03	45.36
		1.0	0.16	0.01	0.0008	0.00‡	0.06	249.44
		0.1	0.19	0.01	0.0007	0.01	0.10	453.45
	0.1	10.0	0.16†	0.00†† ‡	0.0083	0.02†††	-	24.54
		1.0	0.28†	0.01††	0.0048	0.04†††	-	44.61
		0.1	0.28†	0.01††	0.0043	0.05†††	-	48.58
	0.01	10.0	0.25	0.01	0.0643	0.01	0.12	3.72
		1.0	0.26	0.01	0.0623	0.01	0.12	4.06
		0.1	0.26	0.01	0.0620	0.01	0.12	4.09
1.0	1.0	10.0	0.05†	0.00†† ‡	0.0020	0.01†††	-	45.35
		1.0	0.17†	0.01††	0.0008	0.02†††	-	249.39
		0.1	0.20†	0.01††	0.0007	0.06†††	-	453.34
	0.1	10.0	0.15†	0.00†† ‡	0.0085	0.02†††	-	24.49
		1.0	0.29†	0.01††	0.0050	0.04†††	-	44.52
		0.1	0.29†	0.01††	0.0044	0.05†††	-	48.48
	0.01	10.0	0.29†	0.01††	0.0647	0.05†††	-	3.63
		1.0	0.29†	0.02††	0.0625	0.04†††	-	3.96
		0.1	0.29†	0.02††	0.0621	0.04†††	-	3.99

(*) The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

† $h = (0.1)h_{op}$

†† $h = (0.5)h_{op}$

††† $h = (2.8)h_{op}$

‡ No change in the first four digets.

Table 11. Inspection Intervals for Maximum Income, Continuous Screening and B = 0.0001, C_{S1} = 100.0

A	λ(failures/ hour)	α_o	(0.115) $\cdot h_{op}$	(0.535) $\cdot h_{op}$	Inspection Interval	(2.0) $\cdot h_{op}$	(5.0) $\cdot h_{op}$	$-TRI_1^*$
		(hours)	(%) (*)	(%) (*)	(hours)	(%) (*)	(%) (*)	
0.5	1.0	10.0	0.15	0.00‡	0.0043	0.01	0.07	9.04
		1.0	0.36	0.02	0.0019	0.02	0.15	49.70
		0.1	0.47	0.02	0.0014	0.03	0.21	90.33
	0.1	10.0	0.46†	0.02††	0.0166	0.07†††	-	4.74
		1.0	0.55†	0.02††	0.0140	0.09†††	-	8.62
		0.1	0.55†	0.02††	0.0137	0.10†††	-	9.39
	0.01	10.0	1.35	0.08	0.1012	0.09	0.63	0.45
		1.0	1.36	0.08	0.1002	0.10	0.64	0.49
		0.1	1.36	0.08	0.1000	0.10	0.64	0.50
0.9	1.0	10.0	0.15	0.00‡	0.0045	0.01	0.07	9.00
		1.0	0.33	0.01	0.0020	0.02	0.07	49.50
		0.1	0.47	0.02	0.0014	0.03	0.21	89.97
	0.1	10.0	0.43†	0.02††	0.0165	0.07†††	-	4.54
		1.0	0.58†	0.03††	0.0125	0.10†††	-	8.26
		0.1	0.63†	0.03††	0.0120	0.10†††	-	9.00
	0.01	10.0	5.33	0.31	0.1296	0.39	2.50	0.09
		1.0	5.35	0.31	0.1292	0.39	2.50	0.10
		0.1	5.35	0.31	0.1292	0.39	2.51	0.10
1.0	1.0	10.0	0.15†	0.00†† ‡	0.0046	0.02†††	-	9.00
		1.0	0.37†	0.01††	0.0020	0.07†††	-	49.45
		0.1	0.50†	0.02††	0.0014	0.08†††	-	89.88
	0.1	10.0	0.46†	0.02††	0.0158	0.06†††	-	4.49
		1.0	0.61†	0.03††	0.0117	0.08†††	-	8.17
		0.1	0.61†	0.03††	0.0112	0.09†††	-	9.00
	0.01	10.0	404.98†	28.99††	0.1319	58.58†††	-	-0.001
		1.0	412.69†	29.94††	0.1311	59.71†††	-	-0.001
		0.1	412.69†	29.94††	0.1311	59.71†††	-	-0.001

(*)The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

† $h = (0.1)h_{op}$

†† $h = (0.5)h_{op}$

††† $h = (2.8)h_{op}$

‡ No change in the first four digets.

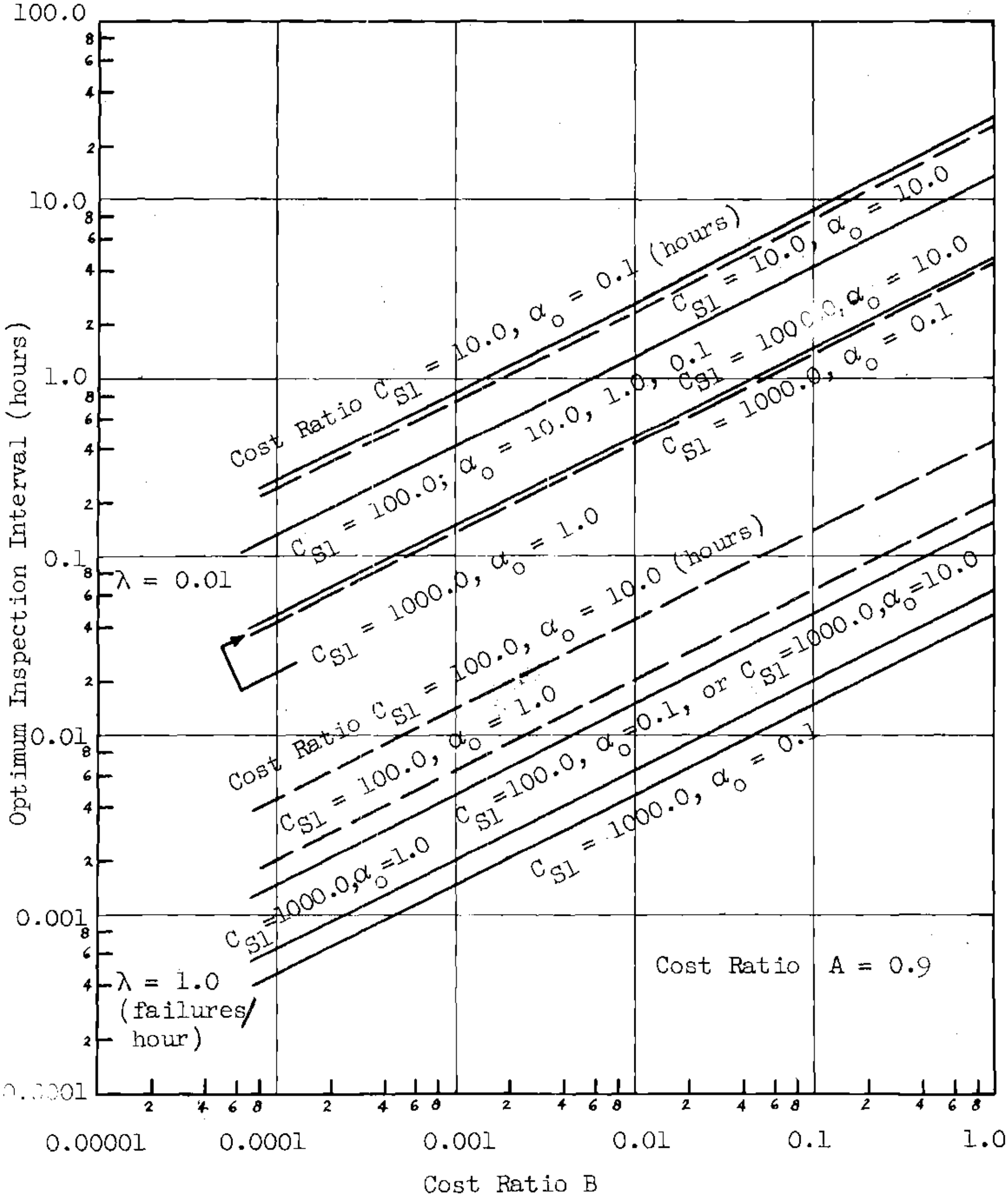

Figure 5. Optimum Inspection Intervals for Maximum Income - Continuous Screening.

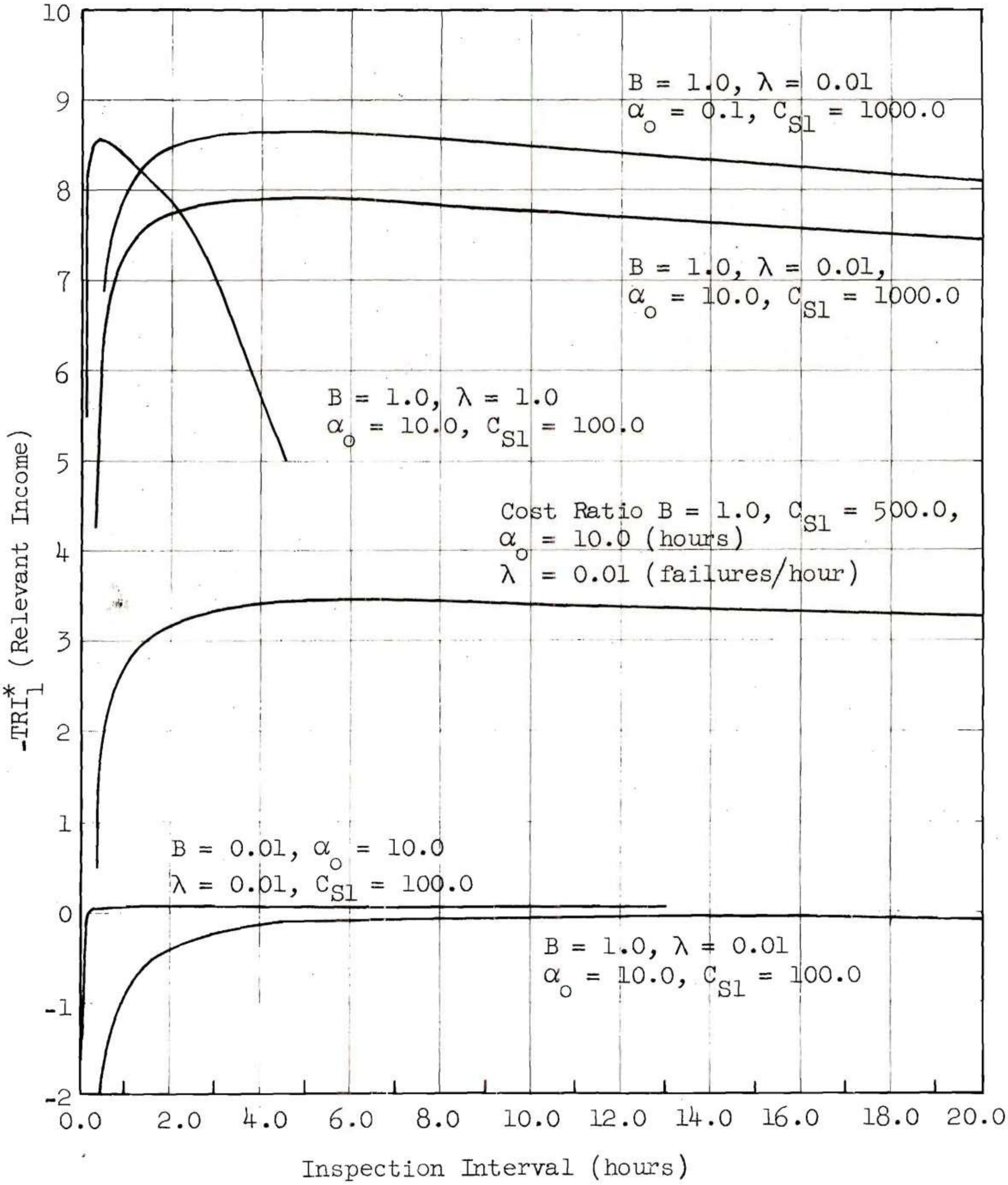

Figure 6. Total Relevant Income Equation - Continuous Screening and Cost Ratio A = 0.9.

APPENDIX G

INSPECTION INTERVALS FOR MINIMUM COST-CONTINUOUS SCREENING AND CHANCE FAILURES

The Tables in this Appendix give the optimum inspection interval (h_{op}) and TRC_1^* ($h = h_{op}$) for selected values of the cost ratios A and B, and λ. The Tables also contain the per cent increase in TRC_1^* (and hence $E(v)$) for four changes in h about h_{op}. For example, if B = 0.01 (see Table 15) and A = 0.90, λ = 0.01, then h_{op} = 1.346, TRC_1^* (h = 1.346) = 91.49 and

$$TRC_1^* \ (h = (0.115)(1.346)) = (91.49)(1.0 + 0.0553),$$

$$TRC_1^* \ (h = (0.535)(1.346)) = (91.49)(1.0 + 0.0032),$$

$$TRC_1^* \ (h = (2.0)(1.346)) = (91.49)(1.0 + 0.0040) \ ,$$

$$TRC_1^* \ (h = (5.0)(1.346)) = (91.49)(1.0 + 0.0262).$$

Table 12. Inspection Intervals for Minimum Cost, Continuous Screening and B = 100.0

A	λ(failures / failure free running hour)	(0.1) ·h_{op} (%) (*)	(0.5) ·h_{op} (%) (*)	Inspection Interval (hours)	(2.0) ·h_{op} (%) (*)	(5.0) ·h_{op} (%) (*)	TRC_1^*
0.00	10.0	98.88	2.79	0.69	0.58	2.64	100.79
	1.0	156.13	7.27	4.62	3.45	16.57	105.62
	0.1	242.02	14.35	24.78	12.18	67.16	133.94
	0.01	326.07	20.10	96.24	19.96	125.55	258.04
	0.001	375.43	23.17	314.92	23.17	148.25	685.08
0.25	10.0	98.89	2.79	0.69	0.58	2.64	100.79
	1.0	155.92	7.28	4.63	3.45	16.60	105.63
	0.1	238.32	14.24	25.11	12.24	67.69	134.50
	0.01	296.70	18.43	100.26	18.74	121.02	272.75
	0.001	270.75	16.76	333.86	16.95	110.69	896.75
0.50	10.0	98.91	2.79	0.69	0.58	2.64	100.79
	1.0	155.69	7.28	4.64	3.46	16.63	105.64
	0.1	234.77	14.13	25.44	12.30	68.24	135.05
	0.01	270.72	16.97	104.65	17.76	117.76	287.09
	0.001	205.92	12.80	356.27	13.15	88.22	1,106.13
0.70	10.0	98.92	2.79	0.69	0.58	2.64	100.79
	1.0	155.50	7.28	4.65	3.46	16.66	105.65
	0.1	232.02	14.05	25.71	12.35	68.69	135.48
	0.01	252.06	15.95	108.44	17.12	116.00	298.30
	0.001	169.29	10.57	377.40	11.05	76.20	1,271.71
0.80	10.0	98.93	2.79	0.69	0.58	2.64	100.79
	1.0	155.43	7.28	4.65	3.46	16.67	105.65
	0.1	230.69	14.01	25.84	12.38	68.91	135.69
	0.01	243.36	15.48	110.43	16.85	115.38	303.80
	0.001	154.30	9.67	389.28	10.21	71.55	1,353.76
0.90	10.0	98.94	2.80	0.69	0.58	2.64	100.79
	1.0	155.34	7.28	4.66	3.47	16.68	105.66
	0.1	229.37	13.98	25.98	12.41	69.15	135.90
	0.01	235.06	15.03	112.49	16.61	114.92	309.24
	0.001	141.00	8.87	402.17	9.48	67.63	1,435.28
0.95	10.0	98.82	2.79	0.69	0.58	2.64	100.79
	1.0	155.29	7.28	4.66	3.47	16.68	105.66
	0.1	228.72	13.96	26.04	12.42	69.26	136.00
	0.01	231.05	14.82	113.55	16.50	114.76	311.94
	0.001	134.89	8.50	409.04	9.15	65.92	1,475.83
1.00	10.0	98.82	2.79	0.69	0.58	2.64	100.79
	1.0	155.24	7.28	4.66	3.47	16.69	105.66
	0.1	228.08	13.94	26.11	12.44	69.38	136.11
	0.01	227.14	14.62	114.62	16.40	114.63	314.62
	0.001	129.11	8.15	416.22	8.84	64.36	1,475.83

(*)The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

Table 13. Inspection Intervals for Minimum Cost, Continuous Screening and B = 1.0

A	λ(failures/failure free running hour)	(0.1) ·h_{op}	(0.5) ·h_{op}	Inspection Interval	(1.5) ·h_{op}	(1.9) ·h_{op}	TRC_1^*
		(%) (*)	(%) (*)	(hours)	(%) (*)	(%) (*)	
0.00	10.0	242.37	14.41	0.25	4.24	10.45	1.34
	1.0	326.16	20.11	0.96	6.66	17.02	2.58
	0.1	375.45	23.17	3.15	7.72	19.75	6.85
	0.01	395.11	24.38	10.00	8.13	20.79	20.51
	0.001	401.82	24.80	31.62	8.26	21.14	63.75
0.25	10.0	238.25	14.22	0.25	4.28	10.53	1.34
	1.0	296.75	18.44	1.00	6.21	15.96	2.73
	0.1	270.72	16.75	3.34	5.63	14.44	8.97
	0.01	171.82	10.61	10.66	3.54	9.08	44.24
	0.001	77.44	4.78	33.78	1.59	4.08	309.70
0.50	10.0	235.00	14.16	0.25	4.27	10.55	1.35
	1.0	270.74	16.98	1.05	5.84	15.10	2.87
	0.1	205.90	12.80	3.56	4.34	11.19	11.06
	0.01	104.11	6.44	11.47	2.16	5.54	67.90
	0.001	40.03	2.47	36.44	0.82	2.11	555.33
0.70	10.0	231.82	14.02	0.26	4.31	10.64	1.35
	1.0	252.09	15.95	1.08	5.59	14.54	2.98
	0.1	169.28	10.57	3.77	3.63	9.39	12.72
	0.01	76.25	4.72	12.26	1.59	4.09	86.72
	0.001	27.58	1.70	39.08	0.57	1.46	751.60
0.80	10.0	230.61	14.00	0.26	4.31	10.65	1.36
	1.0	243.33	15.47	1.10	5.49	14.31	3.04
	0.1	154.31	9.67	3.89	3.34	8.67	13.54
	0.01	66.33	4.11	12.72	1.39	3.57	96.11
	0.001	23.47	1.45	40.64	0.48	1.24	849.60
0.90	10.0	229.42	13.98	0.26	4.31	10.66	1.36
	1.0	234.99	15.02	1.12	5.39	14.10	3.09
	0.1	141.00	8.86	4.02	3.09	8.05	14.35
	0.01	58.11	3.61	13.24	1.22	3.15	105.50
	0.001	20.17	1.24	42.39	0.41	1.07	947.54
0.95	10.0	228.83	13.98	0.26	4.30	10.66	1.36
	1.0	231.03	14.82	1.14	5.34	14.00	3.12
	0.1	134.90	8.50	4.09	2.97	7.76	14.80
	0.01	54.50	3.39	13.52	1.15	2.96	110.20
	0.001	18.76	1.16	43.36	0.38	1.00	996.50
1.00	10.0	228.25	13.97	0.26	4.30	10.67	1.36
	1.0	227.20	14.62	1.14	5.29	13.89	3.15
	0.1	129.11	8.15	4.16	2.87	7.50	15.16
	0.01	51.17	3.18	13.82	1.08	2.79	114.82
	0.001	17.46	1.08	44.39	0.36	0.93	1,045.40

(*) The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

Table 14. Inspection Intervals for Minimum Cost, Continuous Screening and B = 0.1.

A	λ(failures / failure free running hours)	(0.1) ·h_{op} (%) (*)	(0.5) ·h_{op} (%) (*)	Inspection Interval (hours)	(1.5) ·h_{op} (%) (*)	(1.9) ·h_{op} (%) (*)	TRC_1^*
0.00	10.0	326.53	20.17	0.10	6.63	16.96	0.26
	1.0	375.85	23.23	0.31	7.68	19.69	0.69
	0.1	395.19	24.40	1.00	8.12	20.78	2.05
	0.01	401.85	24.80	3.16	8.26	21.14	6.37
	0.001	403.97	24.93	10.00	8.31	21.26	20.05
0.25	10.0	296.20	18.35	0.10	6.26	16.04	0.27
	1.0	270.45	16.71	0.33	5.65	14.48	0.90
	0.1	171.79	10.61	1.06	3.55	9.08	4.42
	0.01	77.43	4.78	3.38	1.59	4.08	30.97
	0.001	28.20	1.74	10.69	0.58	1.48	268.76
0.50	10.0	270.64	16.96	0.10	5.85	15.12	0.29
	1.0	206.01	12.81	0.36	4.33	11.18	1.11
	0.1	104.13	6.44	1.15	2.15	5.53	6.79
	0.01	40.03	2.47	3.64	0.82	2.11	55.53
	0.001	13.57	0.83	11.54	0.27	0.71	517.38
0.70	10.0	252.35	16.00	0.11	5.57	14.50	0.30
	1.0	169.23	10.57	0.38	3.63	9.40	1.27
	0.1	76.27	4.73	1.23	1.59	4.08	8.67
	0.01	27.58	1.70	3.91	0.57	1.46	75.16
	0.001	9.12	0.56	12.39	0.18	0.48	716.18
0.80	10.0	244.07	15.59	0.11	5.42	14.19	0.30
	1.0	154.38	9.68	0.39	3.33	8.66	1.35
	0.1	66.33	4.11	1.28	1.39	3.57	9.61
	0.01	23.46	1.45	4.06	0.48	1.24	84.96
	0.001	7.70	0.47	12.89	0.15	0.40	815.55
0.90	10.0	234.76	15.00	0.11	5.41	14.14	0.31
	1.0	141.11	8.88	0.40	3.08	8.03	1.44
	0.1	58.12	3.61	1.32	1.22	3.15	10.55
	0.01	20.17	1.24	4.24	0.41	1.07	94.75
	0.001	6.57	0.40	13.46	0.13	0.34	914.90
0.95	10.0	230.35	14.71	0.11	5.41	14.11	0.31
	1.0	134.81	8.49	0.41	2.98	7.77	1.48
	0.1	54.51	3.39	1.35	1.15	2.96	11.02
	0.01	18.76	1.16	4.34	0.38	1.00	99.65
	0.001	6.09	0.37	13.77	0.12	0.32	964.56
1.00	10.0	227.56	14.68	0.11	5.25	13.83	0.31
	1.0	129.21	8.17	0.42	2.86	7.48	1.52
	0.1	51.17	3.18	1.38	1.08	2.79	11.48
	0.01	17.46	1.08	4.44	0.36	0.93	104.54
	0.001	5.66	0.35	14.11	0.11	0.29	1,014.21

(*)The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

Table 15. Inspection Intervals for Minimum Cost, Continuous Screening and B = 0.01

A	λ(failures / failures free running hour)	(0.115) ·h_{op} (%) (*)	(0.535) ·h_{op} (%) (*)	Inspection Interval (hours)	(2.0) ·h_{op} (%) (*)	(5.0) ·h_{op} (%) (*)	TRC_1^*
0.00	10.0	315.46	18.69	0.031	23.20	148.37	0.07
	1.0	331.96	19.67	0.100	24.43	156.24	0.20
	0.1	337.82	20.04	0.316	24.80	158.76	0.64
	0.01	339.68	20.15	1.000	24.93	159.59	2.00
	0.001	340.25	20.19	3.162	24.98	159.87	6.33
0.25	10.0	228.07	13.61	0.033	16.88	110.46	0.09
	1.0	144.47	8.58	0.106	10.65	68.71	0.44
	0.1	65.11	3.86	0.338	4.78	30.71	3.10
	0.01	23.71	1.40	1.069	1.74	11.15	26.88
	0.001	7.87	0.46	3.380	0.57	3.70	255.92
0.50	10.0	173.18	10.35	0.036	13.15	88.23	0.11
	1.0	87.55	5.21	0.114	6.50	42.38	0.68
	0.1	33.65	2.00	0.365	2.48	15.97	5.55
	0.01	11.40	0.67	1.154	0.83	5.37	51.74
	0.001	3.69	0.21	3.651	0.27	1.73	505.48
0.70	10.0	142.29	8.54	0.038	11.07	76.28	0.13
	1.0	64.13	3.82	0.123	4.80	31.69	0.87
	0.1	23.19	1.37	0.391	1.71	11.08	7.52
	0.01	7.67	0.45	1.239	0.56	3.62	71.62
	0.001	2.46	0.14	3.921	0.18	1.15	705.10
0.80	10.0	129.57	7.78	0.039	10.25	71.72	0.14
	1.0	55.80	3.33	0.127	4.19	27.93	0.96
	0.1	19.73	1.17	0.406	1.46	9.47	8.50
	0.01	6.47	0.38	1.289	0.47	3.06	81.56
	0.001	2.07	0.12	4.080	0.15	0.97	804.90
90	10.0	118.68	7.18	0.040	9.47	67.60	0.14
	1.0	48.87	2.92	0.132	3.70	24.88	1.05
	0.1	16.96	1.00	0.424	1.25	8.19	9.48
	0.01	5.53	0.32	1.346	0.40	2.62	91.49
	0.001	1.76	0.10	4.262	0.12	0.83	904.70
95	10.0	113.28	6.85	0.041	9.19	66.09	0.15
	1.0	45.86	2.74	0.135	3.48	23.54	1.10
	0.1	15.77	0.93	0.434	1.17	7.64	9.96
	0.01	5.12	0.30	1.377	0.37	2.43	96.46
	0.001	1.63	0.09	4.361	0.12	0.77	954.59
100	10.0	108.79	6.62	0.042	8.81	64.25	0.15
	1.0	43.03	2.57	0.138	3.29	22.36	1.15
	0.1	14.68	0.87	0.444	1.09	7.14	10.45
	0.01	4.76	0.28	1.411	0.35	2.26	101.42
	0.001	1.51	0.09	4.469	0.11	0.71	1,004.48

(*)The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

Table 16. Inspection Intervals for Minimum Cost, Continuous Screening and B = 0.001

A	λ(failures/failure free running hour)	(0.115) $\cdot h_{op}$ (%) (*)	(0.535) $\cdot h_{op}$ (%) (*)	Inspection Interval (hours)	(2.0) $\cdot h_{op}$ (%) (*)	(5.0) $\cdot h_{op}$ (%) (*)	TRC_1^*
0.00	10.0	332.28	19.72	0.010	24.38	156.06	0.02
	1.0	337.31	19.96	0.032	24.89	159.04	0.06
	0.1	339.59	20.14	0.100	24.95	159.65	0.20
	0.01	340.24	20.18	0.316	24.98	159.88	0.63
	0.001	340.45	20.20	1.000	24.99	159.95	2.00
0.25	10.0	145.39	8.72	0.011	10.49	68.18	0.40
	1.0	65.06	3.85	0.034	4.79	30.74	0.31
	0.1	23.70	1.40	0.107	1.74	11.15	2.69
	0.01	7.87	0.46	0.338	0.57	3.70	25.59
	0.001	2.52	0.15	1.069	0.18	1.18	251.87
0.50	10.0	87.04	5.13	0.011	6.59	42.68	0.07
	1.0	33.71	2.00	0.036	2.47	15.94	0.56
	0.1	11.40	0.67	0.115	0.83	5.37	5.17
	0.01	3.69	0.21	0.365	0.27	1.73	50.55
	0.001	1.17	0.06	1.154	0.08	0.55	501.73
0.70	10.0	63.98	3.80	0.012	4.82	31.78	0.09
	1.0	23.15	1.37	0.039	1.72	11.10	0.75
	0.1	7.68	0.45	0.124	0.56	3.62	7.16
	0.01	2.46	0.14	0.392	0.18	1.15	70.51
	0.001	0.78	0.04	1.240	0.05	0.36	701.61
0.80	10.0	55.68	3.31	0.013	4.21	28.01	0.10
	1.0	19.72	1.17	0.041	1.46	9.48	0.85
	0.1	6.48	0.38	0.129	0.47	3.06	8.16
	0.01	2.07	0.12	0.408	0.15	0.97	80.49
	0.001	0.65	0.03	1.291	0.04	0.30	801.55
0.90	10.0	49.05	2.94	0.013	3.67	24.77	0.10
	1.0	17.00	1.01	0.042	1.25	8.17	0.95
	0.1	5.52	0.32	0.135	0.40	2.62	9.15
	0.01	1.76	0.10	0.426	0.12	0.83	90.47
	0.001	0.56	0.03	1.348	0.04	0.26	901.48
0.95	10.0	45.90	2.75	0.014	3.47	23.52	0.11
	1.0	15.75	0.93	0.043	1.17	7.65	1.00
	0.1	5.12	0.30	0.138	0.37	2.43	9.64
	0.01	1.63	0.09	0.436	0.12	0.77	95.46
	0.001	0.51	0.03	1.380	0.03	0.24	951.45
1.00	10.0	43.06	2.58	0.014	3.28	22.34	0.11
	1.0	14.71	0.87	0.044	1.08	7.13	1.04
	0.1	4.76	0.28	0.141	0.35	2.26	10.14
	0.01	1.51	0.09	0.447	0.11	0.71	100.45
	0.001	0.48	0.02	1.414	0.03	0.22	1,001.41

(*) The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

Table 17. Inspection Intervals for Minimum Cost, Continuous Screening and B = 0.0001.

A	λ(failures/ failure free running hour)	(0.115) ·h_{op} (%) (*)	(0.535) ·h_{op} (%) (*)	Inspection Interval (hours)	(2.0) ·h_{op} (%) (*)	(5.0) ·h_{op} (%) (*)	TRC_1^*
0.00	10.0	342.47	20.76	0.0031	23.99	156.16	0.006
	1.0	339.91	20.19	0.0100	24.89	159.47	0.02
	0.1	339.73	20.10	0.0317	25.07	160.17	0.06
	0.01	340.35	20.18	0.1000	25.00	160.01	0.20
	0.001	340.49	20.20	0.3162	25.00	160.00	0.63
0.25	10.0	63.78	3.65	0.0034	5.02	31.47	0.03
	1.0	23.51	1.37	0.0108	1.77	11.26	0.26
	0.1	7.87	0.46	0.0338	0.57	3.69	2.56
	0.01	2.52	0.15	0.1069	0.18	1.18	25.19
	0.001	0.80	0.04	0.3380	0.05	0.37	250.59
0.50	10.0	34.34	2.10	0.0036	2.36	15.58	0.06
	1.0	11.43	0.68	0.0115	0.83	5.36	0.52
	0.1	3.68	0.21	0.0365	0.27	1.73	5.05
	0.01	1.17	0.06	0.1154	0.08	0.55	50.17
	0.001	0.37	0.02	0.3651	0.02	0.17	500.55
0.70	10.0	23.32	1.39	0.0039	1.69	11.00	0.08
	1.0	7.64	0.45	0.0124	0.57	3.64	0.72
	0.1	2.46	0.14	0.0391	0.18	1.15	7.05
	0.01	0.78	0.04	0.1240	0.05	0.36	70.16
	0.001	0.24	0.01	0.3922	0.01	0.11	700.51
0.80	10.0	19.86	1.19	0.0040	1.43	9.40	0.08
	1.0	6.47	0.38	0.0129	0.47	3.06	0.82
	0.1	2.07	0.12	0.0408	0.15	0.97	8.05
	0.01	0.65	0.03	0.1290	0.04	0.30	80.15
	0.001	0.20	0.01	0.4082	0.01	0.09	800.49
0.90	10.0	17.18	1.04	0.0042	1.22	8.06	0.09
	1.0	5.50	0.32	0.0135	0.41	2.63	0.91
	0.1	1.76	0.10	0.0426	0.13	0.83	9.05
	0.01	0.56	0.03	0.1348	0.04	0.26	90.15
	0.001	0.17	0.01	0.4264	0.01	0.08	900.47
0.95	10.0	15.71	0.92	0.0043	1.18	7.68	0.10
	1.0	5.11	0.30	0.0138	0.38	2.44	0.96
	0.1	1.63	0.09	0.0436	0.11	0.76	9.54
	0.01	0.51	0.03	0.1380	0.03	0.24	95.14
	0.001	0.16	0.00(**)	0.4365	0.01	0.07	950.45
1.00	10.0	14.43	0.83	0.0045	1.13	7.30	0.10
	1.0	4.75	0.28	0.0141	0.35	2.26	1.01
	0.1	1.51	0.09	0.0446	0.11	0.71	10.04
	0.01	0.48	0.02	0.1414	0.03	0.22	100.14
	0.001	0.15	0.00(**)	0.4472	0.01	0.07	1,000.45

(*) The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

(**) No change in the first six places.

Table 18. Inspection Intervals for Minimum Cost, Continuous Screening and B = 0.00001

A	λ(failures / failure free running hour)	(0.115) ·h op (%) (*)	(0.535) ·h op (%) (*)	Inspection Interval (hours)	(2.0) ·h op (%) (*)	(5.0) ·h op (%) (*)	TRC^*_1
0.00	10.0	343.19	20.70	0.0010	24.32	157.63	0.002
	1.0	344.92	20.91	0.0031	24.16	157.26	0.006
	0.1	340.68	20.23	0.0100	24.95	159.82	0.02
	0.01	339.97	20.12	0.0317	25.09	160.28	0.06
	0.001	340.43	20.19	0.1000	25.01	160.04	0.20
0.25	10.0	21.71	1.09	0.0011	2.10	12.32	0.03
	1.0	7.71	0.44	0.0034	0.60	3.78	0.26
	0.1	2.50	0.14	0.0108	0.18	1.19	2.52
	0.01	0.80	0.04	0.0338	0.05	0.37	25.06
	0.001	0.25	0.01	0.1069	0.01	0.11	250.19
0.50	10.0	11.52	0.69	0.0011	0.81	5.31	0.05
	1.0	3.77	0.23	0.0036	0.25	1.68	0.50
	0.1	1.17	0.07	0.0115	0.08	0.55	5.02
	0.01	0.37	0.02	0.0365	0.02	0.17	50.05
	0.001	0.11	0.00(**)	0.1154	0.00(**)	0.05	500.19
0.70	10.0	7.24	0.38	0.0013	0.64	3.88	0.07
	1.0	2.48	0.14	0.0039	0.17	1.14	0.70
	0.1	0.78	0.04	0.0124	0.05	0.36	7.02
	0.01	0.24	0.01	0.0393	0.01	0.11	70.05
	0.001	0.07	0.00(**)	0.1240	0.00(**)	0.03	700.16
0.80	10.0	6.42	0.37	0.0013	0.48	3.09	0.08
	1.0	2.09	0.12	0.0040	0.14	0.96	0.80
	0.1	0.65	0.03	0.0129	0.04	0.30	8.02
	0.01	0.20	0.01	0.0408	0.01	0.09	80.05
	0.001	0.06	0.00(**)	0.1290	0.00(**)	0.03	800.15
0.90	10.0	5.79	0.36	0.0013	0.36	2.47	0.09
	1.0	1.80	0.11	0.0042	0.12	0.81	0.90
	0.1	0.55	0.03	0.0135	0.04	0.26	9.01
	0.01	0.17	0.01	0.0426	0.01	0.08	90.05
	0.001	0.05	0.00(**)	0.1348	0.00(**)	0.02	900.15
0.95	10.0	4.80	0.25	0.0014	0.43	2.62	0.10
	1.0	1.64	0.09	0.0043	0.11	0.76	0.95
	0.1	0.51	0.03	0.0138	0.03	0.24	9.51
	0.01	0.16	0.00(**)	0.0437	0.01	0.07	95.04
	0.001	0.05	0.00(**)	0.1380	0.00(**)	0.02	950.14
1.00	10.0	4.60	0.25	0.0014	0.37	2.35	0.10
	1.0	1.50	0.08	0.0045	0.11	0.72	1.00
	0.1	0.48	0.02	0.0141	0.03	0.22	10.01
	0.01	0.15	0.00(**)	0.0448	0.01	0.07	100.04
	0.001	0.04	0.00(**)	0.1414	0.00(**)	0.02	1,000.14

(*)The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

(**)No change in the first six places.

Table 19. Inspection Intervals for Minimum Cost, Continuous Screening and B = 0.000001

A	λ(failures/ failure free running hour)	(0.115) h_{op} (%) (*)	(0.535) h_{op} (%) (*)	Inspection Interval (hours)	(2.0) h_{op} (%) (*)	(5.0) h_{op} (%) (*)	TRC_1^*
0.00	10.0	260.78	7.83	0.0004	38.87	204.33	0.0006
	1.0	343.96	20.75	0.0010	24.37	157.98	0.002
	0.1	345.17	20.93	0.0031	24.18	157.37	0.006
	0.01	340.75	20.24	0.0100	24.95	159.86	0.02
	0.001	340.00	20.12	0.0317	25.09	160.29	0.06
0.25	10.0	6.72	0.28	0.0004	0.79	4.39	0.02
	1.0	2.31	0.11	0.0011	0.22	1.31	0.25
	0.1	0.78	0.04	0.0034	0.06	0.38	2.50
	0.01	0.25	0.01	0.0108	0.01	0.12	25.02
	0.001	0.08	0.00(**)	0.0338	0.00(**)	0.03	250.06
0.50	10.0	3.48	0.18	0.0004	0.30	1.85	0.05
	1.0	1.18	0.07	0.0011	0.08	0.54	0.50
	0.1	0.38	0.02	0.0036	0.02	0.17	5.00
	0.01	0.11	0.00(**)	0.0115	0.00(**)	0.05	50.02
	0.001	0.03	0.00(**)	0.0365	0.00(**)	0.01	500.05
0.70	10.0	2.54	0.15	0.0004	0.16	1.11	0.07
	1.0	0.73	0.03	0.0013	0.06	0.39	0.70
	0.1	0.25	0.01	0.0039	0.01	0.11	7.00
	0.01	0.07	0.00(**)	0.0124	0.00(**)	0.03	70.02
	0.001	0.02	0.00(**)	0.0391	0.00(**)	0.01	700.05
0.80	10.0	2.25	0.15	0.0004	0.12	0.87	0.08
	1.0	0.65	0.03	0.0012	0.04	0.31	0.80
	0.1	0.21	0.01	0.0040	0.01	0.09	8.00
	0.01	0.06	0.00(**)	0.0129	0.00(**)	0.03	80.02
	0.001	0.02	0.00(**)	0.0408	0.00(**)	0.00(**)	800.05
0.90	10.0	2.02	0.14	0.0004	0.08	0.69	0.09
	1.0	0.58	0.03	0.0013	0.03	0.24	0.90
	0.1	0.18	0.01	0.0042	0.01	0.08	9.00
	0.01	0.05	0.00(**)	0.0135	0.00(**)	0.02	90.01
	0.001	0.01	0.00(**)	0.0426	0.00(**)	0.00(**)	900.05
0.95	10.0	1.92	0.14	0.0004	0.07	0.62	0.10
	1.0	0.48	0.02	0.0014	0.04	0.26	0.95
	0.1	0.16	0.00(**)	0.0043	0.01	0.07	9.50
	0.01	0.05	0.00(**)	0.0138	0.00(**)	0.02	95.01
	0.001	0.01	0.00(**)	0.0436	0.00(**)	0.00(**)	950.05
1.00	10.0	1.84	0.13	0.0004	0.05	0.55	0.10
	1.0	0.46	0.02	0.0014	0.03	0.23	1.00
	0.1	0.15	0.00(**)	0.0045	0.01	0.07	10.00
	0.01	0.04	0.00(**)	0.0141	0.00(**)	0.02	100.01
	0.001	0.01	0.00(**)	0.0448	0.00(**)	0.00(**)	1,000.05

(*)The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

(**)No change in the first six places.

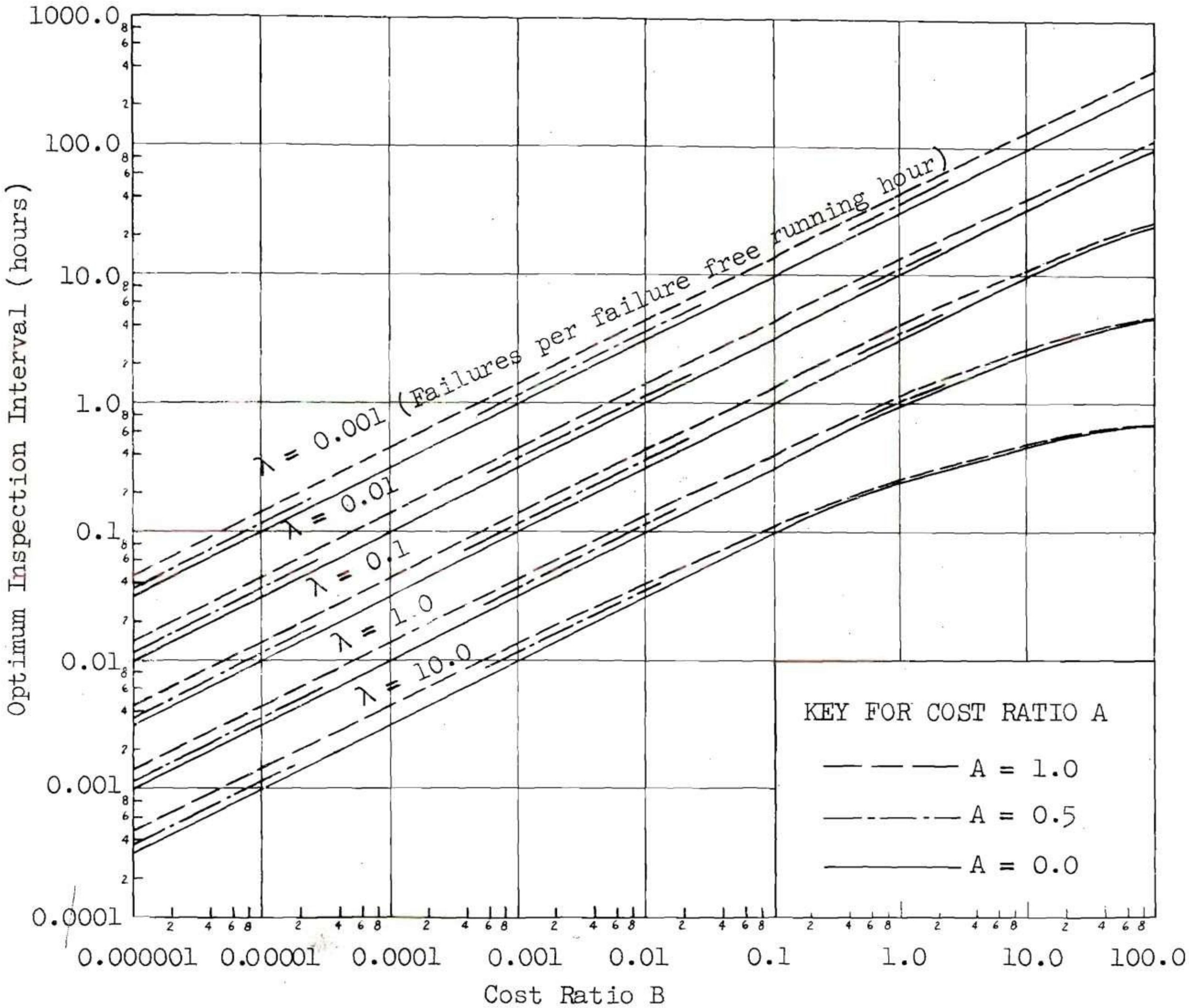

Figure 7. Optimum Inspection Intervals for Minimum Cost - Continuous Screening.

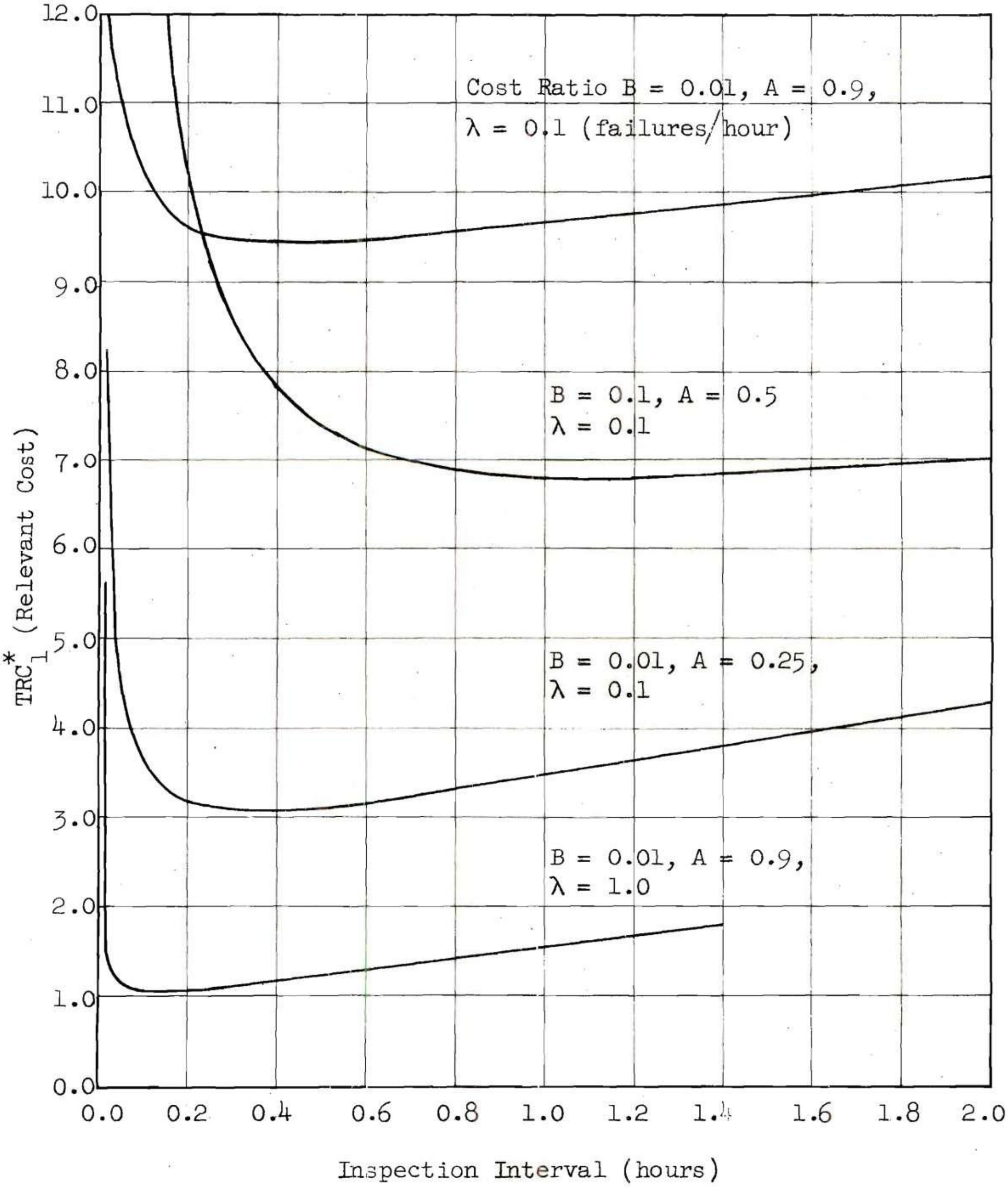

Figure 8. Total Relevant Cost Equation - Continuous Screening.

APPENDIX H

INSPECTION INTERVALS FOR MINIMUM COST - SEQUENTIAL SCREENING AND CHANCE FAILURES

The Tables in this Appendix give the optimum inspection interval (h_{op}) and TRC_2^* ($h = h_{op}$) for selected values of λ, R, and the cost ratios A and B, with d = 0. The Tables also contain TRC_2^* for four values of h: h_a, a = k-2, k, k+1, and k+3, where

$$h_a = \frac{2^a}{R},$$

and

$$k = \left[\!\left[\frac{\log (Rh_{op})}{\log 2}\right]\!\right].$$

For example, if B = 0.01 (see Table 23) and R = 100, λ = 0.1, A = 0.50 then h_{op} = 0.437, TRC_2^* (h = 0.437) = 10.48, k = 5, and

$$TRC_2^* \ (h_{k-2} = 0.080) = 11.31,$$

$$TRC_2^* \ (h_k = 0.320) = 10.50,$$

$$TRC_2^* \ (h_{k+1} = 0.640) = 10.51,$$

$$TRC_2^* \ (h_{k+3} = 2.560) = 11.42 .$$

Table 20. Inspection Intervals for Minimum Cost, Sequential Screening and B = 100.0, R = 10 (units/hour)

A	λ(failures/failure free running hour)	$h_{k-2}^{(*)}$	$h_{k}^{(*)}$	Inspection Interval (hours)	$h_{k+1}^{(*)}$	$h_{k+3}^{(*)}$	TRC_2^*
0.25	1.0	183.27	107.96	4.64	107.03	126.20	106.07
	0.1	225.54	136.72	26.08	152.78	305.62	136.71
	0.01	556.70	316.58	114.57	350.76	920.43	315.38
0.50	1.0	183.20	107.83	4.64	106.88	126.00	105.93
	0.1	225.39	136.52	26.09	152.56	305.35	136.51
	0.01	556.50	316.33	114.59	350.49	920.10	315.12
0.90	1.0	183.08	107.64	4.66	106.64	125.68	105.71
	0.1	225.15	136.20	26.10	152.20	304.91	136.19
	0.01	556.18	315.93	114.61	350.05	919.58	314.72

(*) The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

Table 21. Inspection Intervals for Minimum Cost, Sequential Screening, B = 1.0.

A	λ(failures/ failure free running hour)	R (units/ hour)	$h_{k-2}^{(*)}$	$h_{k}^{(*)}$	Inspection Interval (hours)	$h_{k+1}^{(*)}$	$h_{k+3}^{(*)}$	TRC_2^*
0.25	1.0	10	6.69	3.49	1.09	3.56	7.86	3.40
		100	7.88	3.51	1.14	3.21	6.22	3.20
	0.1	10	23.64	15.71	4.08	16.10	29.43	15.56
		100	26.50	15.82	4.15	15.34	24.74	15.23
	0.01	10	133.74	115.38	13.74	118.37	162.10	115.35
		100	140.91	115.56	13.81	116.07	148.38	114.89
0.50	1.0	10	6.67	3.42	1.10	3.46	7.71	3.32
		100	7.86	3.50	1.14	3.19	6.20	3.18
	0.1	10	23.56	15.59	4.11	15.95	29.23	15.43
		100	26.48	15.80	4.16	15.32	24.72	15.20
	0.01	10	133.61	115.21	13.77	118.17	161.85	115.17
		100	140.89	115.53	13.81	116.04	148.34	114.87
0.90	1.0	10	6.63	3.30	1.14	3.30	7.47	3.18
		100	7.85	3.48	1.14	3.16	6.16	3.15
	0.1	10	23.44	15.39	4.15	15.71	28.91	15.22
		100	26.46	15.77	4.16	15.28	24.67	15.17
	0.01	10	133.41	114.93	13.81	117.85	161.45	114.89
		100	140.86	115.49	13.82	116.00	148.29	114.83

(*)The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

Table 22. Inspection Intervals for Minimum Cost, Sequential Screening and B = 0.1, R = 10 (units/hour)

A	λ(failures/ failure free running hour)	$h_{k-2}^{(*)}$	$h_k^{(*)}$	Inspection Interval (hours)	$h_{k+1}^{(*)}$	$h_{k+3}^{(*)}$	TRC_2^*
0.25	1.0	2.10(**)	1.73	0.33	1.67	2.43	1.65
	0.1	15.22	11.93	1.27	11.80	14.20	11.76
	0.01	113.18	105.16	4.34	105.30	114.39	104.94
0.50	1.0	2.10(**)	1.70	0.35	1.62	2.33	1.61
	0.1	15.20	11.86	1.31	11.70	14.05	11.66
	0.01	113.10	105.03	4.37	105.15	114.19	104.81
0.90	1.0	2.10	1.54	0.41	1.66	3.49	1.54
	0.1	15.16	11.74	1.36	11.54	13.81	11.52
	0.01	112.98	104.83	4.42	104.91	113.87	104.59

(*)The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

(**)$h = h_{k-1}$

Table 23. Inspection Intervals for Minimum Cost, Sequential Screening, B = 0.01

A	λ(failures/ failure free running hour)	R (units/ hour)	$h_{k-2}^{(*)}$	$h_{k}^{(*)}$	Inspection Interval (hours)	$h_{k+1}^{(*)}$	$h_{k+3}^{(*)}$	TRC_2^*
0.25	1.0	100	1.52	1.19	0.127	1.18	1.42	1.18
		1000	1.34	1.15	0.137	1.18	1.62	1.15
	0.1	100	11.32	10.52	0.434	10.53	11.44	10.49
		1000	11.60	10.53	0.443	10.46	11.12	10.46
	0.01	100	103.33	101.48	1.403	101.74	105.38	101.47
		1000	104.04	101.50	1.410	101.53	104.29	101.43
0.50	1.0	100	1.52	1.18	0.131	1.17	1.40	1.17
		1000	1.34	1.15	0.138	1.18	1.62	1.15
	0.1	100	11.31	10.50	0.437	10.51	11.42	10.48
		1000	11.60	10.53	0.443	10.46	11.12	10.46
	0.01	100	103.32	101.46	1.405	101.72	105.36	101.46
		1000	104.04	101.50	1.410	101.53	104.28	101.43
0.90	1.0	100	1.52	1.17	0.137	1.15	1.38	1.15
		1000	1.33	1.15	0.138	1.18	1.61	1.15
	0.1	100	11.30	10.48	0.442	10.49	11.39	10.46
		1000	11.60	10.52	0.444	10.46	11.11	10.45
	0.01	100	103.30	101.43	1.410	101.69	105.32	101.43
		1000	104.04	101.50	1.411	101.52	104.28	101.42

(*)The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

Table 24. Inspection Intervals for Minimum Cost, Sequential Screening and B = 0.001, R = 100 (units/hour)

A	λ(failures / failure free running hour)	$h_{k-2}^{(*)}$	$h_k^{(*)}$	Inspection Interval (hours)	$h_{k+1}^{(*)}$	$h_{k+3}^{(*)}$	TRC_2^*
0.25	1.0	1.10(**)	1.07	0.034	1.06	1.12	1.06
	0.1	10.52	10.19	0.130	10.17	10.38	10.17
	0.01	100.31	100.51	0.437	100.52	101.38	100.49
0.50	1.0	1.10(**)	1.06	0.036	1.06	1.11	1.06
	0.1	10.52	10.18	0.133	10.16	10.37	10.16
	0.01	101.30	100.50	0.440	100.51	101.36	100.47
0.90	1.0	1.10	1.05	0.043	1.05	1.18	1.05
	0.1	10.51	10.17	0.139	10.15	10.34	10.14
	0.01	101.29	100.48	0.446	100.48	101.33	100.45

(*) The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

(**) $h = h_{k-1}$.

Table 25. Inspection Intervals for Minimum Cost, Sequential Screening and B = 0.0001
R = 1000 (units/hour)

A	λ(failures / failure free running hour)	$h_{k-2}^{(*)}$	$h_{k}^{(*)}$	Inspection Interval (hours)	$h_{k+1}^{(*)}$	$h_{k+3}^{(*)}$	TRC_2^*
0.25	1.0	1.05	1.02	0.013	1.02	1.04	1.02
	0.1	10.13	10.05	0.044	10.05	10.14	10.05
	0.01	100.33	100.15	0.140	100.17	100.53	100.15
0.50	1.0	1.05	1.02	0.013	1.02	1.04	1.01
	0.1	10.13	10.05	0.044	10.05	10.14	10.05
	0.01	100.33	100.14	0.141	100.17	100.53	100.14
0.90	1.0	1.05	1.02	0.014	1.01	1.03	1.01
	0.1	10.13	10.05	0.044	10.05	10.13	10.04
	0.01	100.33	100.14	0.141	100.17	100.52	100.14

(*) The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

Table 26. Inspection Intervals for Minimum Cost, Sequential Screening and B = 0.00001, R = 1000 (units/hour)

A	λ(failures/failure free running hour)	$h_{k-2}^{(*)}$	$h_k^{(*)}$	Inspection Interval (hours)	$h_{k+1}^{(*)}$	$h_{k+3}^{(*)}$	TRC_2^*
0.25	1.0	1.01(**)	1.01	0.0034	1.01	1.01	1.00
	0.1	10.05	10.02	0.0130	10.02	10.04	10.02
	0.01	100.13	100.05	0.0437	100.05	100.14	100.05
0.50	1.0	1.01(**)	1.01	0.0036	1.00	1.01	1.00
	0.1	10.05	10.02	0.0133	10.02	10.04	10.02
	0.01	100.13	100.05	0.0440	100.05	100.14	100.05
0.90	1.0	1.01	1.00	0.0043	1.00	1.02	1.00
	0.1	10.05	10.02	0.0140	10.01	10.03	10.01
	0.01	100.13	100.05	0.0446	100.05	100.13	100.04

(*) The discussion at the beginning of this Appendix explains the meaning of the numbers in this column.

(**) $h = h_{k-1}$.

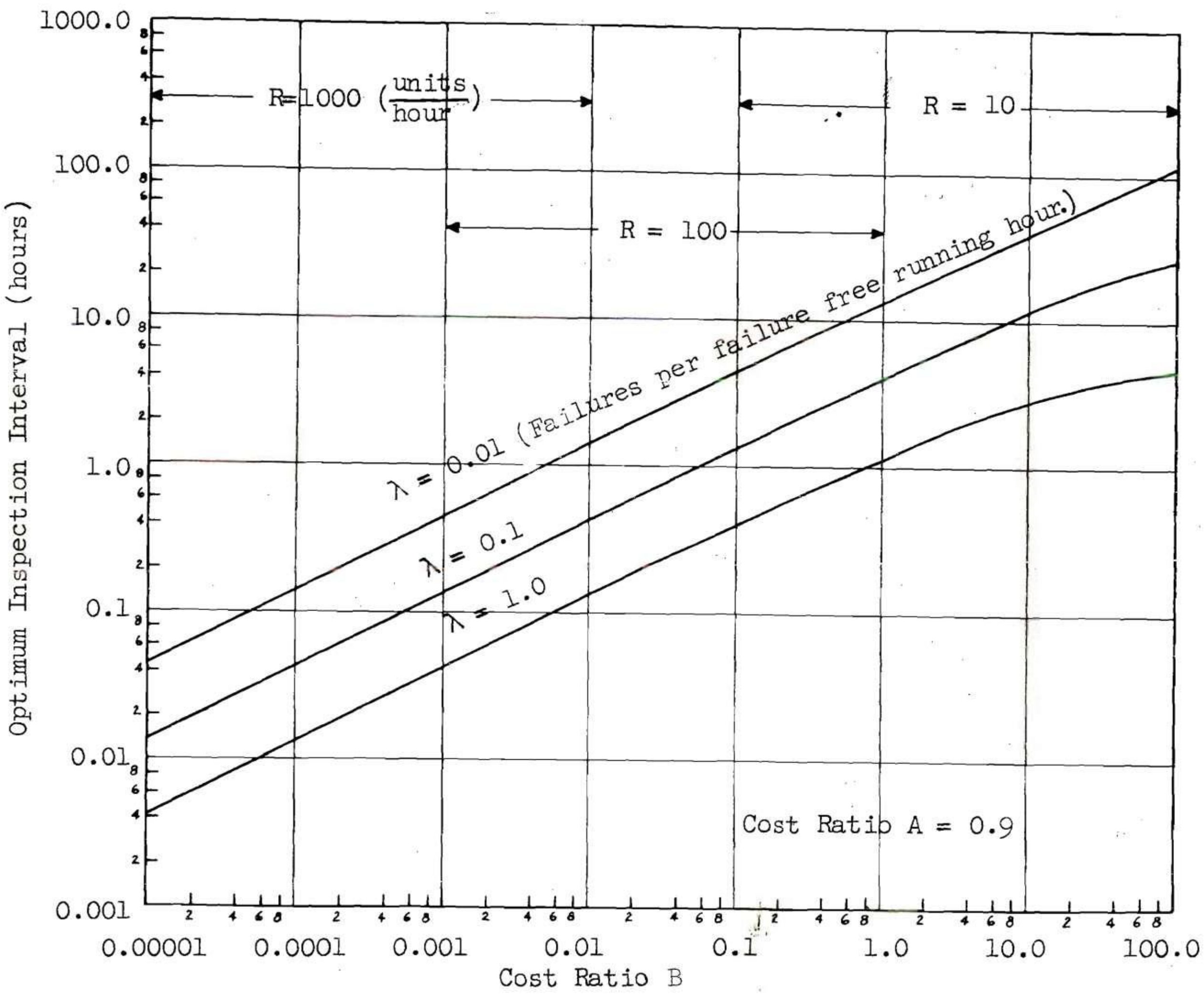

Figure 9. Optimum Inspection Intervals for Minimum Cost - Sequential Screening.

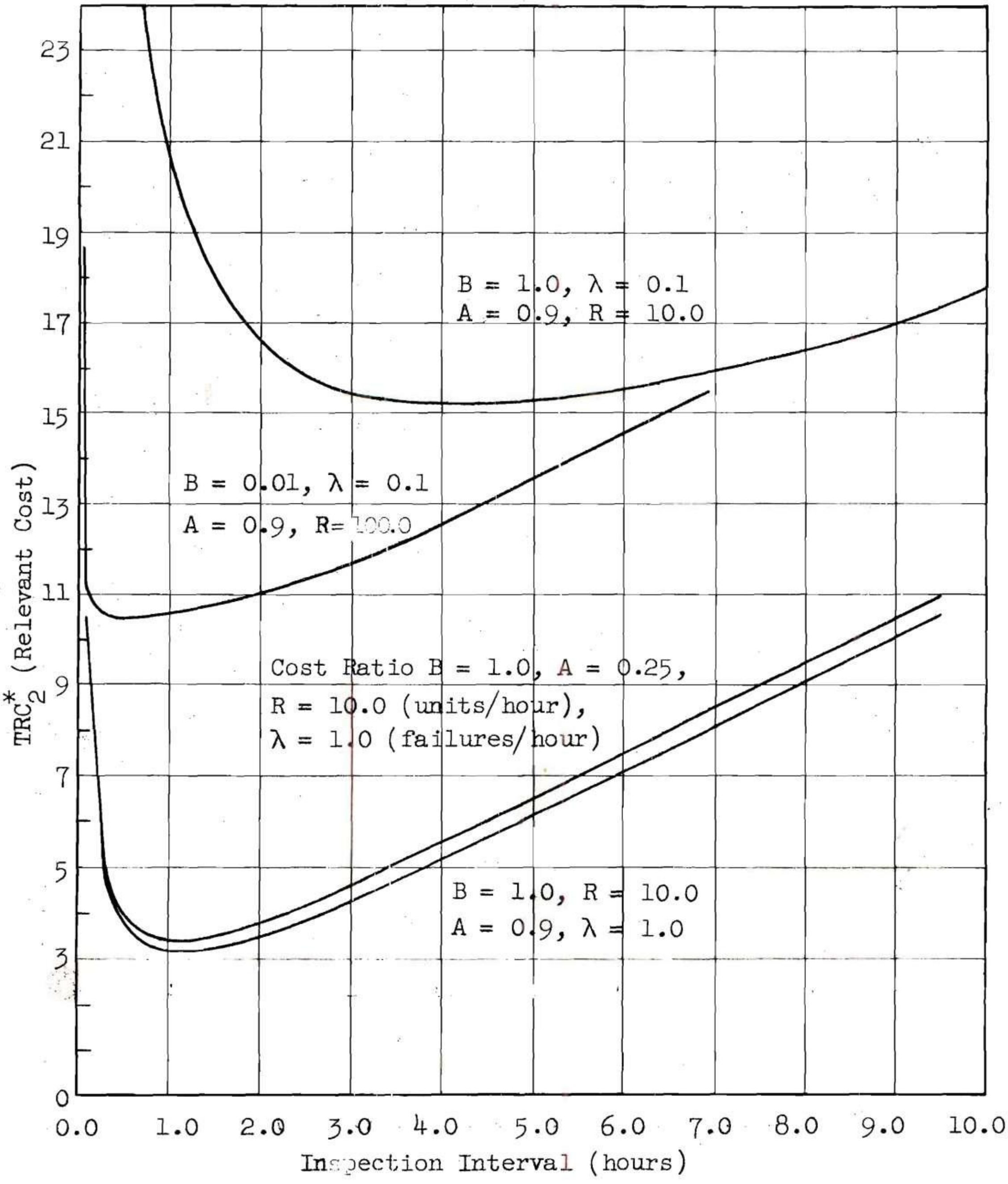

Figure 10. Total Relevant Cost Equation - Sequential Screening.

APPENDIX I

THE PROCESS FAILURE DENSITY FUNCTION OF MODEL II

Wearout Models

The selection of a failure density function for wearout failure will be confined to two parameter distributions. It is felt that the quality of life data obtainable in most practical cases never warrants the use of more than two parameters. A survey of the literature on wearout models shows that there are only four models of general importance. The wearout models are reviewed first, then the process failure density function (the "addition" of the gamma and exponential densities) is discussed.

Weibull Distribution

The two parameter Weibull probability density function is

$$f(t) = \begin{cases} \alpha\lambda t^{\alpha-1}\, e^{-\lambda t^{\alpha}} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}, \tag{1}$$

where $\alpha > 0$ and $\lambda > 0$. When $\alpha = 1$ it is seen that the exponential distribution is a special case of this distribution. The Weibull distribution has been investigated as a model for wearout failure in (11), (12), and (18).

Truncated Normal Distribution

If $f(t)$ is the normal probability density function ($N(\mu,\sigma)$, μ the mean and σ^2 the variance, $\mu > 0$) then the truncated normal density function, $f^*(t)$, is

$$f^*(t) = \begin{cases} \dfrac{f(t)}{\int_0^\infty f(X)dX} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases} \tag{2}$$

The truncation is necessary because $f(t)$ is defined for $t < 0$. The truncation is of negligible importance if $\mu - 3\sigma > 0$, and then $f(t)$ may be used for $f^*(t)$. Flehinger and Lewis (11) and Pieruschka (13) study the use of this distribution is wearout models.

<u>Lognormal Distribution</u>

Pieruschka (13) makes the substitution

$$t - u = \mu \ln (t/\mu)$$

in the normal distribution which produces the lognormal distribution. The lognormal density function is

$$f(t) = \begin{cases} \dfrac{\mu}{(2\pi)^{0.5}\sigma t} \cdot e^{-\frac{1}{2}(\frac{\mu}{\sigma})^2 \ln^2 (\frac{t}{\mu})} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases} \tag{3}$$

Pieruschka, in (13), states "Experience indicates that the normal distribution must be replaced by the logarithmic normal distribution when the failures are distributed over time." But, Flehinger and Lewis, in (11), show that the lognormal distribution is not capable of fitting as wide a range of lifetime behavior as the truncated normal, Weibull, or gamma distributions. The reader is referred to (25) for a thorough account of the applications of the lognormal distribution.

Gamma Distribution

The gamma distribution has been proposed as a wearout failure model in (13), (14), and (18). The gamma density function is

$$f(t) = \begin{cases} \dfrac{\mu^N t^{N-1} e^{-\mu t}}{\Gamma(N)} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases} , \tag{4}$$

and $\mu > 0$, $N > 0$.

Lloyd and Lipow (18) show that under a realistic set of assumptions that a process that fails because of chance and wearout causes, the underlying distribution for wearout alone is the gamma distribution. They assume a failure occurs because of wearout, say fatigue or depletion of material, after receiving N "shocks." The time between "shocks" is exponentially distributed, or the arrival of a "shock" is a random occurrence. There are two types of "shocks." One type of "shock" will cause immediate failure and this corresponds to chance failure. The mean time between large "shocks," or those that produce immediate failure, is λ occurrences per hour. The other type of "shock", call it the mild "shock," causes failure only after N such shocks have been registered. This mild "shock" corresponds to partial wearout. The parameter N can be interpreted as the processes' resistance to mild "shocks," or in a redundant system, the number of components in parallel. The parameter μ is the average arrival rate of mild "shocks." If t is the time to a process failure, and N is an integer, it is shown in (18) that the probability density function of t, g(t), is

$$g(t) = \begin{cases} e^{-\lambda t} \left\{ \dfrac{\mu^N t^{N-1} e^{-\mu t}}{\Gamma(N)} + \lambda \left[1 - \displaystyle\int_0^t \frac{\mu^N X^{N-1} e^{-\mu X}}{\Gamma(N)} \, dX \right] \right\} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases} . \quad (5)$$

Exactly the same form for $g(t)$ is obtained by the "addition" (see equation (3) of Appendix A) of the exponential and gamma densities.

Process Failure Density Function

Equation (5) may be simplified to the form

$$g(t) = \begin{cases} e^{-(\lambda+\mu)t} \left[\dfrac{\mu^N t^{N-1}}{(N-1)!} + \displaystyle\sum_{i=0}^{N-1} \frac{\lambda(\mu t)^i}{i!} \right] & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases} . \quad (6)$$

If $G(t)$ is the process failure distribution function then

$$G(t) = 1 - e^{-\lambda t} \left(1 - \int_0^t \frac{\mu^N X^{N-1} e^{-\mu X}}{(N-1)!} \, dX \right)$$

$$= 1 - e^{-(\lambda+\mu)t} \sum_{i=0}^{N-1} \frac{(\mu t)^i}{i!} . \quad (7)$$

In order to simplify the calculation of values of $g(t)$ and $G(t)$ for any λ, μ, and N, equations (6) and (7) are rewritten in the forms (see (18))

$$g(t) = e^{-\lambda t} \left\{ \mu q(N-1, \mu t) + \lambda \left[1 - P(N, \mu t) \right] \right\}, \quad (8)$$

and

$$G(t) = 1 - e^{-\lambda t} \left(1 - P(N, \mu t) \right) , \quad (9)$$

where

$$q(X, a) = \frac{a^X e^{-a}}{X!} \quad , \tag{10}$$

and

$$P(C, a) = \sum_{X=C}^{\infty} q(X, a) \ . \tag{11}$$

Both equations (10) and (11) have been tabulated in (20). Lloyd and Lipow, in (18), have produced the graph of $g(t)$ for the cases $\mu = 1.0$ and $N = 5$, $\lambda = 0.01$, 0.05, 0.10; $N = 10$, $\lambda = 0.01$, 0.05, 0.10; and $N = 20$, $\lambda = 0.01$, 0.05, 0.10. The general shape of $g(t)$ is similar to the well known human mortality curve.

It may seem that the importance of the normal and lognormal distributions have been neglected. This is not the case because the gamma distribution can be approximated by the normal distribution in a certain limiting sense. If $\lambda = 0$, and $\mu = \beta N$, then as $N \to \infty$ $g(t)$ becomes normally distributed with mean β^{-1} and variance $(\beta^2 N)^{-1}$ (see (18)). In (13) it is noted that the lognormal distribution becomes approximately normal when the mean is much larger than the variance. Therefore, the three distributions are all related in a certain limiting sense. Pieruschka (13) states "We may prove, by a χ^2-test, that a given histogram is fitted by a gamma distribution as well as by a normal distribution." In many cases the failure data will be equally well fitted by any one of the three distributions.

APPENDIX J

ESTIMATING THE PARAMETERS OF THE PROCESS FAILURE DISTRIBUTION FOR MODEL II

Let

t_i = the ith observation on the process running time to a failure,

t_i^* = the ith observation on the process running time to when a failure was discovered.(*)

First assume that the t_i numbers are available.

When the process fails it may be possible, for example by inspecting the machine tool, to determine whether the failure was a result of chance or wearout causes. If this is the case let

t_{ic} = the ith observation on the process running time to a chance failure, and

t_{iw} = the ith observation on the process running time to a wearout failure.

The t_{ic} numbers and equation (1) of Appendix B will provide an estimator for λ. To obtain estimates for μ and N one may use the maximum likelihood method or the technique of matching moments. There is no closed form solution for the maximum likelihood estimators for the gamma distribution and the reader is referred to (18) for an approximate method for obtaining these estimates. The technique of matching moments is to equate

---

(*)It is assumed that the lag time d, which it takes the inspector to decide whether the process has failed or not failed, has been subtracted in obtaining the t_i^* numbers.

population moments with their corresponding unbiased sample moments. For the gamma distribution, this technique of matching moments will usually require less computation than the maximum likelihood method, and Lloyd and Lipow (18) state "... for small samples there is apparently no easy way of choosing between the two ways for this particular problem."

It is usually the case that it is impossible to partition t_i into t_{ic} and t_{iw}. To obtain estimates of λ, μ, and N the technique of matching moments may be used. Now, three moments of g(t) are needed, and it can be shown that if $\bar{\mu}$ and σ^2 are the mean and variance of g(t), then

$$\bar{\mu} = \left\{1 - \left(\frac{\mu}{\lambda+\mu}\right)^N\right\} \frac{1}{\lambda}, \text{ and}$$

$$\sigma^2 = \lambda^{-2}\left\{1 - 2\left(\frac{\mu}{\mu+\lambda}\right)^N \cdot \frac{\lambda N}{\mu+\lambda} - \left(\frac{\mu}{\lambda+\mu}\right)^{2N}\right\}.$$

One further moment must be calculated and then there are three equations with three unknowns which can be solved for λ, μ, and N.

Now assume only the t_i^* numbers are available. If t_i^* can be divided into t_{ic}^* and t_{iw}^*, then equation (5) of Appendix B will give a consistent estimator for λ. When the number of defectives produced is known for each t_{iw}^* value, then t_{iw} can be computed, and the technique of matching moments or the method of maximum likelihood may be used to determine estimates of μ and N.

When the amount of defective production is not known for each t_i^* number and the t_{ic}^* and t_{iw}^* values are unattainable, there does not seem to be an easy way of estimating λ, μ, and N.

APPENDIX K

DERIVATION OF $P(\ell)$ AND $E(I_N|\ell)$, $\ell = 1,2,3,4$, FOR MODEL II

Recall that

$$E(I_N|\ell) = VE(U|\ell) - E(Uv|\ell) - f_X , \tag{1}$$

where

$$E(Uv|\ell) \doteq \frac{H_R E(M|\ell)}{E((L+D)|\ell)} , \tag{2}$$

$$E(U|\ell) \doteq \frac{H_R RE(T|\ell)}{E((L+D)|\ell)} , \tag{3}$$

$$E(M|\ell) = CE(I|\ell) - RE(sH_b|\ell) + E(SN_o|\ell) + E(R_C|\ell)$$
$$+ C_d E(D|\ell) + YRE(L|\ell) , \tag{4}$$

and

$$E((L+D)|\ell) = hE(I|\ell) + d + E(D|\ell) , \tag{5}$$

for $\ell = 1,2,3,4$.

<u>Case of $\ell = 4$</u>

In this case the process does not fail and it is easily seen that (the effect of d is d = 0)

$$E(T|4) = Kh , \tag{6}$$

$$E(L|4) + E(D|4) = Kh + D_W , \tag{7}$$

$$E(I|4) = K , \tag{8}$$

$$E(sH_b|4) = E(SN_o|4) = 0 , \tag{9}$$

$$E(R_C|4) = (R_C)_w . \tag{10}$$

Therefore

$$E(I_N|4) = H_R \left\{ \frac{(V-Y)RKh - (CK + C_d D_w + (R_C)_w)}{Kh + D_w} \right\} - f_X . \tag{11}$$

<u>Case of $\ell = 3$</u>

Consider the salvage income (cost). It is seen that (see Wilks (24) page 61, and equation (3) of Chapter III),

$$E(sH_b|3) = s_w \int_0^{Kh} H_b dG(t|3) =$$

$$= s_w \int_0^{Kh} \frac{H_b \gamma(t) e^{-\lambda t}}{P(\ell=3)} dt . \tag{12}$$

Therefore

$$E(I|3) = \int_0^{Kh} \frac{I\gamma(t)e^{-\lambda t}}{P(\ell=3)} dt , \tag{13}$$

$$E(L|3) = h E (I|3) + d,^{(*)} \tag{14}$$

$$E(SN_o|3) = S_w \rho (R \{h+d\}) , \tag{15}$$

(*) If the process fails in the Kth inspection interval then the effect of d is d = 0. The $E(I_N|3)$ equation is formulated for $d \neq 0$ when the process fails in the Kth inspection interval. The effect of d = 0 for the Kth inspection interval may be incorporated into the model by partitioning $\ell = 3$ into two disjoint events: failure prior to (K-1)h, failure in the interval ((K-1)h, Kh) with d = 0.

$$E(D|3) = D_w , \tag{16}$$

$$E(R_C|3) = (R_C)_w , \tag{17}$$

$$E(T|3) = \int_0^{Kh} \frac{t\gamma(t)e^{-\lambda t}}{P(\ell=3)} \, dt . \tag{18}$$

Expressions for H_b and I are in Appendix D and it follows that

$$E(H_b|3) = hE(I|3) + d - E(T|3) ,$$

therefore

$$E(I_N|3) \doteq H_R \left\{ \frac{(V-s_w)RE(T|3) - \left[\begin{array}{l} (C+Rh(Y-s_w))E(I|3) + \\ S_w \, \rho(R(h+d)) + (R_C)_w \\ + C_d D_w + R(Y-s_w)d \end{array} \right]}{hE(I|3) + d + D_w} \right\} - f_X . \tag{19}$$

<u>Case of $\ell = 1$</u>

In this case there is a different value of s, R_C, D, and S for each chance failure component. First consider salvage income (cost) and

$$E(sH_b|1) = \sum_{q=1}^{n_r} P(q) \, E\left\{ (sH_b | \ell=1) \middle| q \right\} , \tag{20}$$

where q is the event that it was the qth chance failure component that failed, when a chance failure arrived prior to the time (K-1)h.

It is seen that

$$P(q) = \int_0^{(K-1)h} \frac{\lambda_q e^{-\lambda t}}{P(\ell=1)} \left\{ \int_t^{\infty} \gamma(X)dX \right\} dt = \frac{\lambda_q}{\lambda} , \tag{21}$$

hence

$$\sum_{q=1}^{n_r} P(q) = 1 .$$

It follows that

$$E\left\{(sH_b \mid \ell=1) \mid q\right\} = E\left\{sH_b \mid \ell=1 \text{ and } q \text{ occurred}\right\} =$$

$$= \int_0^{(K-1)h} \frac{s_q H_b \lambda_q e^{-\lambda t}}{P(q)P(\ell=1)} \left\{\int_t^\infty \gamma(X)dX\right\} dt ,$$

therefore

$$\sum_{q=1}^{n_r} P(q)E\left\{(sH_b \mid \ell=1) \mid q\right\} = \sum_{q=1}^{n_r} \frac{s_q\lambda_q}{\lambda} \cdot E(H_b \mid \ell=1) . \tag{22}$$

Let

$$\overline{s} = \sum_{q=1}^{n_r} \frac{s_q\lambda_q}{\lambda} . \tag{23}$$

It is evident that the argument outlined above implies the following results:

$$E(D \mid 1) = \overline{D} , \tag{24}$$

$$E(R_C \mid 1) = \overline{R}_C , \tag{25}$$

$$E(SN_o \mid 1) = \overline{S}\rho(R\left\{h+d\right\}) , \tag{26}$$

$$E(I \mid 1) = \int_0^{(K-1)h} \frac{I\lambda e^{-\lambda t}}{P(\ell=1)} \left\{\int_t^\infty \gamma(X)dX\right\} dt , \tag{27}$$

$$E(L \mid 1) = hE(I \mid 1) + d, \tag{28}$$

and

$$E(T|1) = \int_{o}^{(K-1)h} \frac{t\lambda e^{-\lambda t}}{P(\ell=1)} \left\{ \int_{t}^{\infty} \gamma(X)dX \right\} dt \ , \tag{29}$$

where $\overline{D}$, $\overline{R}_C$, and $\overline{S}$ are defined in the same manner as $\overline{s}$ is defined in equation (23). Therefore

$$E(I_N|1) \doteq H_R \left\{ \frac{(V-\overline{s})RE(T|1) - \left[\left\{ C + Rh(Y-\overline{s}) \right\} E(I|1) + \overline{R}_C + C_d\overline{D} + R(Y-\overline{s})d + \overline{S}\,\rho(R(h+d)) \right]}{hE(I|1) + d + \overline{D}} \right\} - f_X . \tag{30}$$

<u>Case of ℓ=2</u>

This case is similar to ℓ=1. It is noticed that (recall that the effect of d is d = 0)

$$E(sH_b|2) = \overline{s}E(H_b|2) = \overline{s}(Kh - E(T|2)), \tag{31}$$

because

$$H_b = Kh - T .$$

Therefore

$$E(I|2) = K , \tag{32}$$

$$E(R_C|2) = \overline{R}_C + (R_C)_w , \tag{33}$$

$$E(SN_o|2) = \overline{S}\rho(R\left\{ h \right\}) , \tag{34}$$

$$E(L|2) = h\ E(I|2) = Kh\ , \tag{35}$$

$$E(T|2) = \int_{(K-1)h}^{Kh} \frac{t\lambda e^{-\lambda t}}{P(\ell=2)} \left\{ \int_t^{\infty} \gamma(X)dX \right\} dt\ , \tag{36}$$

and

$$E(D|2) = D_w\ , \tag{37}$$

because the failed (chance failure) component will be changed at the same time all the wearout components are being changed and $D_w \gg \overline{D}$. Therefore

$$E(I_N|2) \doteq H_R \left\{ \frac{(V-\overline{s})\cdot RE(T|2) - \left[\begin{array}{l} \left\{C + Rh(Y-\overline{s})\right\}K \\ + \overline{R}_C + (R_C^{\prime})_w + C_d D_w \\ + \overline{S}\ \phi(R(h)) \end{array} \right]}{Kh + D_w} \right\} - f_X. \tag{38}$$

<u>The Probabilities $P(\ell)$</u>

When $\ell = 4$, then

$$P(4) = \int_{Kh}^{\infty} g(t)dt = 1 - G(Kh) = e^{-(\lambda+\mu)Kh} \sum_{j=0}^{N-1} \frac{(\mu Kh)^j}{j!}\ , \tag{39}$$

where $G(t)$ is defined by equation (3) of Chapter III.

If $\ell = 3$, it is seen that

$$P(3) = \int_0^{Kh} \gamma(t)e^{-\lambda t}\ dt =$$

$$= \left(\frac{\mu}{\mu+\lambda}\right)^N \left\{1 - e^{-(\lambda+\mu)Kh} \sum_{j=0}^{N-1} \frac{\left[(\lambda+\mu)Kh\right]^j}{j!}\right\}. \tag{40}$$

Now consider the event $\ell = 1$, and it can be shown that

$$P(1) = \int_0^{(K-1)h} \lambda e^{-\lambda t} \left\{\int_t^\infty \gamma(X)dX\right\} dt = \sum_{j=0}^{N-1} \frac{\lambda\mu^j}{j!} \int_0^{(K-1)h} t^j e^{-(\lambda+\mu)t} dt =$$

$$= 1-\left(\frac{\mu}{\lambda+\mu}\right)^N - e^{-(\lambda+\mu)(K-1)h} \sum_{j=0}^{N-1} \frac{\left[(K-1)\mu h\right]^j}{j!} \left\{1 - \left(\frac{\mu}{\lambda+\mu}\right)^{N-j}\right\}. \tag{41}$$

The probability P(2) follows directly from P(1) because $P(\ell=2)$ is $P(\ell=1)$ with K substituted for K-1 minus $P(\ell=1)$.

APPENDIX L

LIST OF SYMBOLS

Model I - Chapter II

h = inspection interval.

λ = number of process failures, as a result of chance causes, per hour.

$e(t)$ = process failure density function, equation (1).

T_M = maximum expected net income.

h_{op} = that $h \geq 0$ that produces T_M.

E = statistical expected value operator.

$[\![\ ]\!]$ = greatest integer function.

V = product sale price.

U = total saleable units produced per year.

H_R = total plant operating clock hours per year.

R = production rate (constant).

s = salvage value per defective.

C = inspection cost.

S = unit screening cost.

"tool cycle" = clock hours from a "new" tool to a "new" tool.

R_C = retooling cost per tool cycle.

D = downtime per tool cycle.

L = total process running hours per tool cycle.

$\bar{L}$ = $E(L)$

I = number of inspections per tool cycle.

$\overline{I}$ = E(I).

H_b = hours of bad production per tool cycle.

$\overline{H}_b$ = $E(H_b)$.

N_o = number of units screened per tool cycle.

$\overline{N}_o$ = $E(N_o)$.

d = time to complete a process inspection.

$\rho(R\{h+d\})$ = mean number of units screened per tool cycle when sequential mid-range search is used, equation (32).

T = time to process failure.

v = total variable cost per non-defective.

Y = total variable production cost per unit produced.

f_x = total fixed process cost per year.

C_d = process idle time cost per hour of idle production.

m = total number of tool cycles in the year.

M = total variable cost per tool cycle.

TRI = total relevant income equation when U is a variable, equation (36).

TRC = total relevant cost equation when U is constant, equation (37).

A = a cost ratio, equation (40).

B = a cost ratio, equation (41).

α_o = d + D.

C_S = a cost ratio for TRI, equation (21).

C_{S1} = a cost ratio for TRI when continuous screening is used, equation (43).

C_{S2} = a cost ratio for TRI when sequential search is used, equation (48).

Model II - Chapter III

R_o = the maximum process running hours, without a wearout failure, to an automatic preventive maintenance shutdown.

$K = R_o/h$, an integer.

h_{op}, K_{op} = the $h \geq 0$, the $K = 1,2,\ldots$ that produce T_M.

$\gamma(t)$ = the process wearout density function, equation (1).

$g(t)$ = the process failure density function, equation (2).

$G(t)$ = the process failure distribution function, equation (3).

λ_w = number of process failures, as a result of wearout, per hour.

N,μ = parameters of $\gamma(t)$, N is an integer and $N/\mu = 1/\lambda_w$.

λ = number of process failures, as a result of chance causes, per hour.

n_r = number of chance failure components.

n_w = number of wearout failure components.

λ_i = number of chance failures per hour, for the ith chance failure component.

s_i = salvage income per defective, when the ith chance failure component failed.

s_w = salvage income per defective, wearout failure.

$$\bar{s} = \sum_{i=1}^{n_r} \lambda_i s_i \Big/ \lambda.$$

D_i = downtime for retooling the ith chance failure component.

D_w = downtime for changing all wearout components.

$$\bar{D} = \sum_{i=1}^{n_r} D_i \lambda_i \Big/ \lambda.$$

R_{C_i} = retooling cost for ith chance failure component.

R_{C_w} = retooling cost for changing all wearout components.

$$\overline{R}_C = \sum_{i=1}^{n_r} R_{C_i}\lambda_i/\lambda \ .$$

S_i = unit screening cost, when the ith chance failure component failed.

S_w = unit screening cost, wearout failure.

$$\overline{S} = \sum_{i=1}^{n_r} S_i\lambda_i/\lambda \ .$$

$P(\ell)$: probability of the event ℓ.

ℓ: any of four disjoint events pertaining to process failure, and

$$\sum_{\ell=1}^{4} P(\ell) = 1.$$

"tool cycle" = time to when a failure is discovered or Kh, whichever occurs first, plus the downtime.

I_N = net income per year.

<u>General Model - Chapter IV</u>

$X(t;\ \xi)$ = distribution of the product quality characteristic, where t is the process running hours measured from the beginning of the tool cycle, and ξ is the measure of the product quality characteristic.

Q = the quality control plan to be used.

Θ = a collection of functionally independent variables each belonging to Q such that the elements of Θ completely determine Q. Θ_{op}, h_{op}, K_{op} = the value of the variables that will produce T_M.

t^* = the time to a process failure, or Kh, whichever occurs first.

"in-control" = the process activity before t^*.

"out-of-control" = the process activity after t^*.

H_o: the hypothesis that the process is in-control.

H_a: the hypothesis that the process is out-of-control.

α = type I error.

β = type II error.

n = sample size.

C_I = cost of type I error.

I_o = number of in-control inspections per tool cycle.

a = fixed cost per sample.

b = variable cost per sample.

BIBLIOGRAPHY

Literature Cited

(1) Duncan, Acheson J., "The Economical Design of $\bar{X}$ Charts Used to Maintain Current Control of a Process," Journal of the American Statistical Association, 51, June 1956, pp. 228-242.

(2) Aroian, Leo A., and Levine, Howard, "The Effectiveness of Quality Control Charts," Journal of the American Statistical Association, 45, 1950, pp. 520-529, Errata, op. cit. 47, 1952, p. 685.

(3) Weiler, H., "On the Most Economical Sample Size for Controlling the Mean of a Population," Annals of Mathematical Statistics, 23, 1952, pp. 247-254.

(4) Weiler, H., "The Use of Runs to Control the Mean in Quality Control," Journal of the American Statistical Association, 48, 1953, pp. 816-825.

(5) Weiler, H., "New Type of Control Chart Limits for Means, Ranges, and Sequential Runs," Journal of the American Statistical Association, 49, 1954, pp. 298-314.

(6) Feigenbaum, A. V., "The Challenge of Total Quality Control," Industrial Quality Control, 13, No. 11, May 1957, pp. 17-23.

(7) Page, E. S., "Control Charts for the Mean of a Normal Population," Journal of the Royal Statistical Society, Series B16, No. 1, 1954, pp. 131-135.

(8) Page, E. S., "Control Charts with Warning Lines," Biometrika, 42, No. 1/2, June 1955, pp. 243-257.

(9) Ito, Shizuo, "On the Characteristics of Control Charts," Statistical Quality Control, 7, May 1956, pp. 299-303.

(10) Fraser, D. A. S., Statistics, An Introduction, New York, John Wiley and Sons, 1958, p. 64.

(11) Flehinger, B. J., and Lewis, P. A., "Two-Parameter Lifetime Distributions for Reliability Studies of Renewal Processes," International Business Machine Journal of Research and Development, 3, No. 1, January 1959, pp. 58-73.

(12) Weibull, W., "A Statistical Distribution Function of Wide Applicability," Journal of Applied Mechanics, 18, 1951, pp. 293-297.

(13) Pieruschka, Erick, "Mathematical Foundation of Reliability Theory, Redstone Arsenal, 1958.

(14) Davis, D. J., "An Analysis of Some Failure Data," Journal of the American Statistical Association, 47, 1952, pp. 113-150.

(15) Feller, William, An Introduction to Probability Theory and its Applications, I, 2nd Ed., New York, John Wiley and Sons, 1958, pp. 411-3.

(16) Moder, Joseph J., "A Sequential Search Procedure for Locating a Response Jump," Technometrics, 4, Nov. 1962, pp. 610-614.

(17) Kiokemeister, Fred L., and Johnson, Richard E., Calculus, with Analytic Geometry, 2nd ed., Boston, Allyn and Bacon, 1960, p. 448.

(18) Lloyd, David K., and Lipow, Myron, Reliability-Management, Methods and Mathematics, Englewood Cliffs, New Jersey, Prentice-Hall, 1962, pp. 145-152.

(19) Pearson, Karl, Tables of the Incomplete Gamma Function, London, Cambridge University Press, 1922.

(20) Molina, E. C., Poisson's Exponential Binomial Limit, Princeton, New Jersey, D. Van Nostrand Company, 1942.

(21) Burington, Richard S., Handbook of Mathematical Tables and Formulas, Sandusky, Ohio, Handbook Publishers, 1950, pp. 7-10.

(22) Cramér, H., Mathematical Methods in Statistics, Princeton, New Jersey, Princeton University Press, 1946, p. 255.

(23) Cochran, W. G., Sampling Techniques, New York, John Wiley and Sons, 1953, p. 128.

(24) Wilks, Samuel S., Mathematical Statistics, New York, John Wiley and Sons, 1962.

(25) Aitchison, J., and Brown, J. A., The Lognormal Distribution, With Special Reference to its Use in Economics, London, Cambridge University Press, 1957.

(26) Goldsmith, P. L., and Whitfield, H., "Average Run Lengths in Cumulative Chart Quality Control Schemes," Technometrics, 3, Feb. 1961, pp. 11-20.

(27) Page, E. S., "Cumulative Sum Charts," Technometrics, 3, Feb. 1961, pp. 1-9.

(28) Moder, Joseph J., "The Role of Mathematical Process Quality Models in Designing Optimal Control Chart Procedures," unpublished paper, School of Industrial Engineering, Georgia Institute of Technology, 1958.

(29) Ray, Dev, Process Quality Models and their Application to Optimizing Quality Control Procedures, unpublished thesis, School of Industrial Engineering, Georgia Institute of Technology, expected 1963.

(30) Bazovsky, Igor, Reliability Theory and Practice, Englewood Cliffs, New Jersey, Prentice-Hall, 1961.

(31) Cowden, Dudley J., Statistical Methods in Quality Control, Englewood Cliffs, New Jersey, Prentice-Hall, 1957, pp. 456-471.

Other References

1. Apostol, Tom M., Mathematical Analysis, Addison-Wesley Publishing Company, Reading, Mass., 1958.

2. Burroughs Corporation, Burroughs Algebraic Compiler, Sales Technical Services, Detroit, Michigan, Bulletin 220-21011-D, January, 1961.

3. Bowker, A. H., and Lieberman, G. J., Engineering Statistics, Prentice-Hall, New Jersey, 1960.

4. Duncan, Acheson J., Quality Control and Industrial Statistics, Richard D. Irwin, Homewood, Illinois, 1959.

5. Hoel, Paul G., Introduction to Mathematical Statistics, 2nd Ed., John Wiley and Sons, New York, 1956.

6. Parzen, Emanuel, Modern Probability Theory and Its Applications, John Wiley and Sons, New York, 1960.